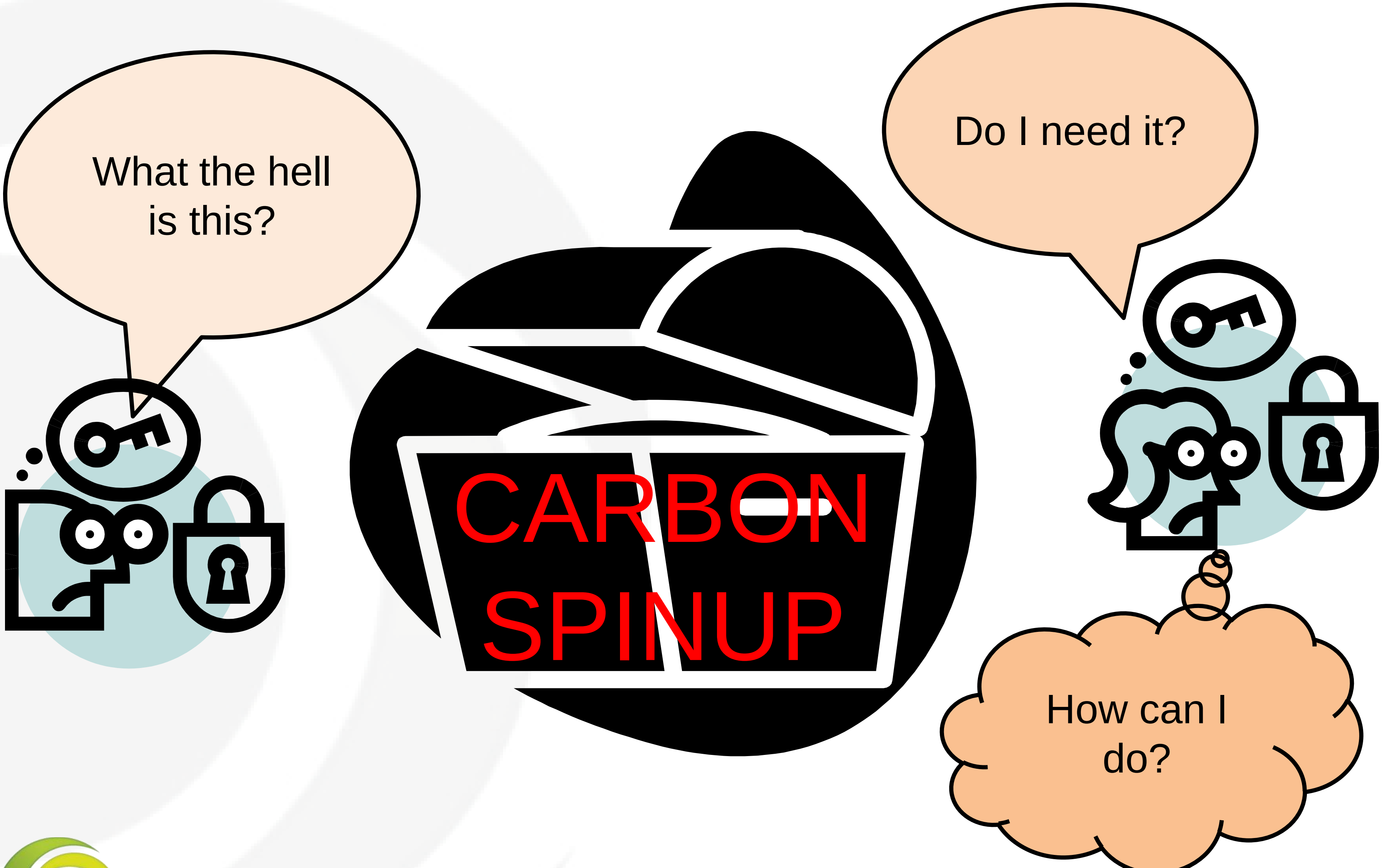
Introduction to the carbon spinup – Equilibrium state – Analytical solution



Carbon Spinup – Analytical solution

Scientific issue – Concepts

Mathematical solution – Estimation of carbon stocks

Code implementation

libIGCM configuration

Implementation done by D. Solyga, N. Vuichard and J. Ghattas

Scientific issue

Use of Terrestrial Biosphere Models?

Steady state equilibrium

Need for a Carbon Spinup

Dynamics of **carbon pools**

Case with two carbon pools

In ORCHIDEE

CENTURY model

What is the use of Terrestrial Biosphere Models?

→ Test our understanding of processes.

→ Lead numerical experiments, for example introduction of disturbances such as:

- Climate Change
- Increasing atmospheric CO_2 concentration
- Land Use Change
- …

➲ A stable initialization state (with no trend) is required to study impacts.

Steady state equilibrium

- Without disturbances, the carbon cycle in terrestrial ecosystems and in TBMs reaches an equilibrium, with all variables at a steady state (constant mean value over a forcing period).

- This steady state equilibrium is usually used as the initialization state of experiments with TBMs (convention in MIPs protocols).

- This ideal state is useful when you don't know the real history (plantation date, fires, LUC, …) so always with regional/global simulations.

- This equilibrium depends on the model (+ parameters), climate, vegetation type, soil.

Need for a Carbon Spinup

The carbon cycle is conceptually represented using **carbon pools** exchanging carbon.

The equilibrium is usually reached by running the carbon model **several thousands of years** to bring all carbon pools at equilibrium.

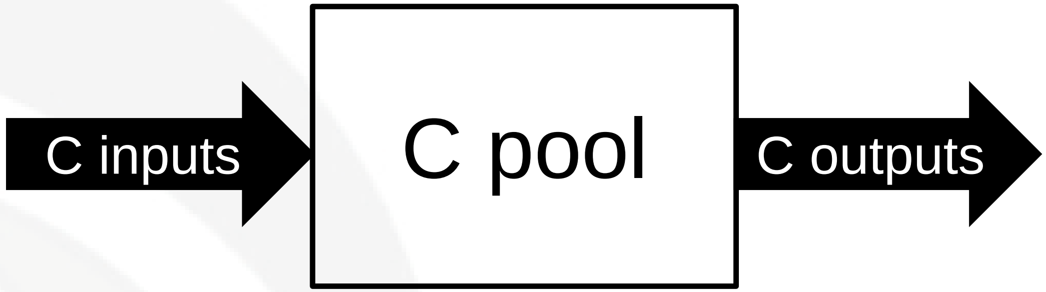
This operation is named as the **spinup** of the model.

The computational cost is quite heavy as compared to the experiment itself (hundreds of years).

➲ We need to optimise the spinup.

➲ Analytical expression of the carbon stocks at the equilibrium state

Carbon processes in terrestrial ecosystems can be represented by **linear first-order differential equations**.



b influx

$$\frac{dC}{dt} = -aC + b$$

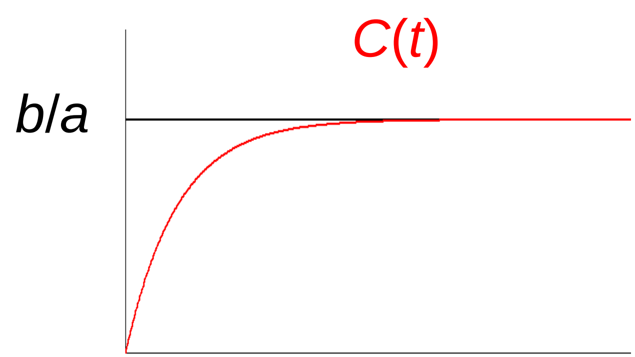
a decomposition rate (a>0)

$a = a_{max}\rho_T\rho_W$

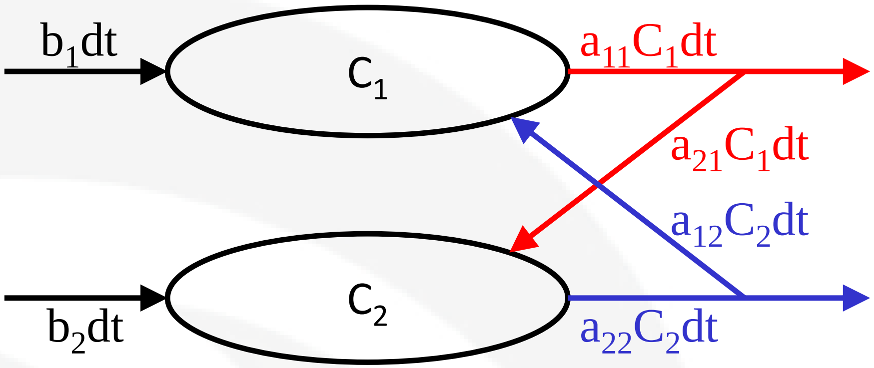
$\tau = 1/a$ residence time

Simple case: one pool, *a* and *b* constants, *C*(*0*)=*0*

$$\Rightarrow \quad C(t) = \frac{b}{a}(1 - e^{-at})$$



Case with two carbon pools



$$\frac{dC_1}{dt} = -a_{11}C_1 + a_{12}C_2 + b_1$$

$$\frac{dC_2}{dt} = a_{21}C_1 + a_{22}C_2 + b_2$$

$$\frac{d}{dt}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -a_{11} & a_{12} \\ a_{21} & -a_{22} \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

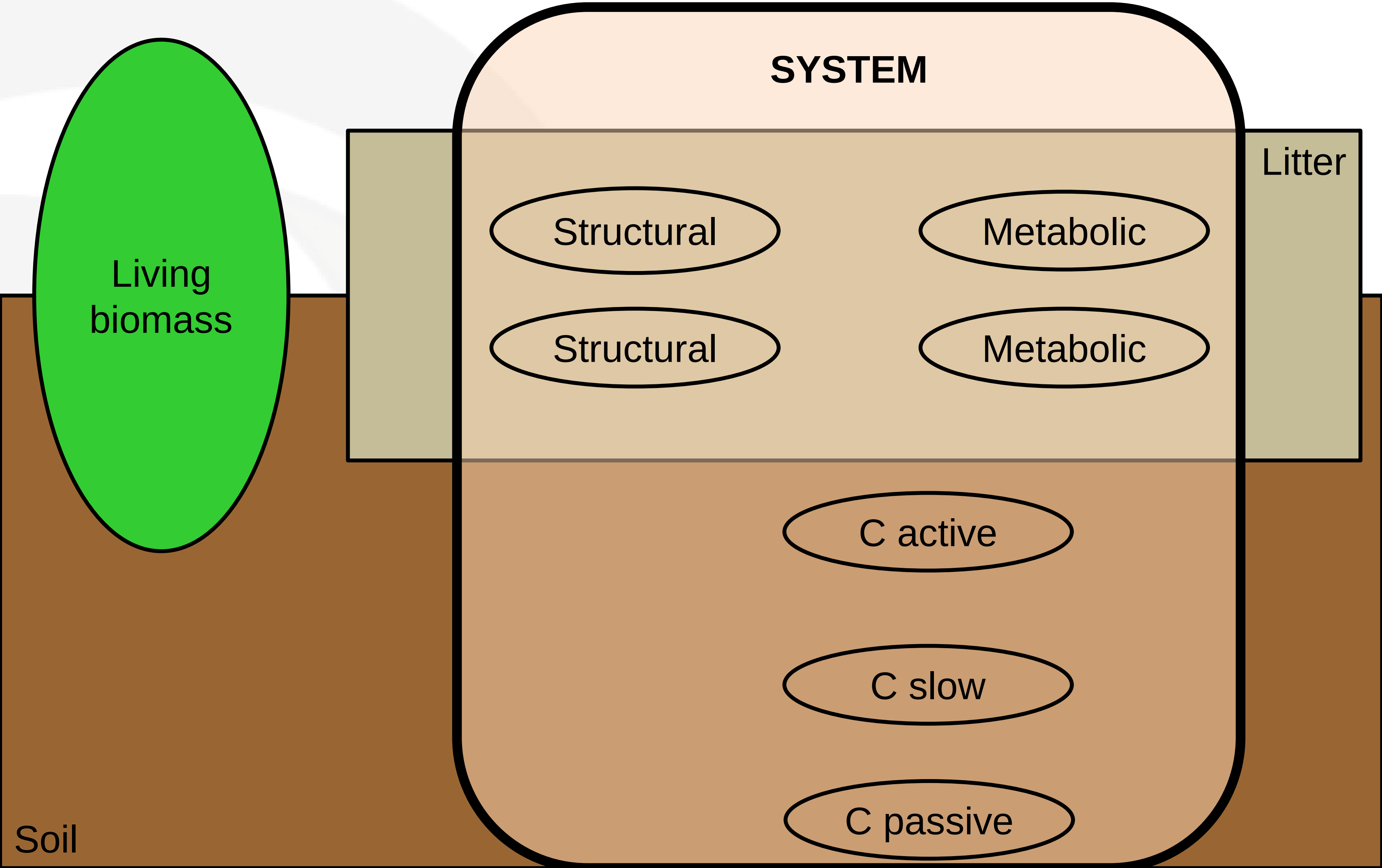
Matrix formulation

Same equation as in the one pool case

$$\frac{dC}{dt} = AC + B$$

➲ Generalization to a vector *C* of any number of carbon pools

In ORCHIDEE



SYSTEM
Living biomass
Litter
Structural
Metabolic
Structural
Metabolic
C active
C slow
C passive
Soil

CENTURY model

1174 SOIL SCI. SOC. AM. .

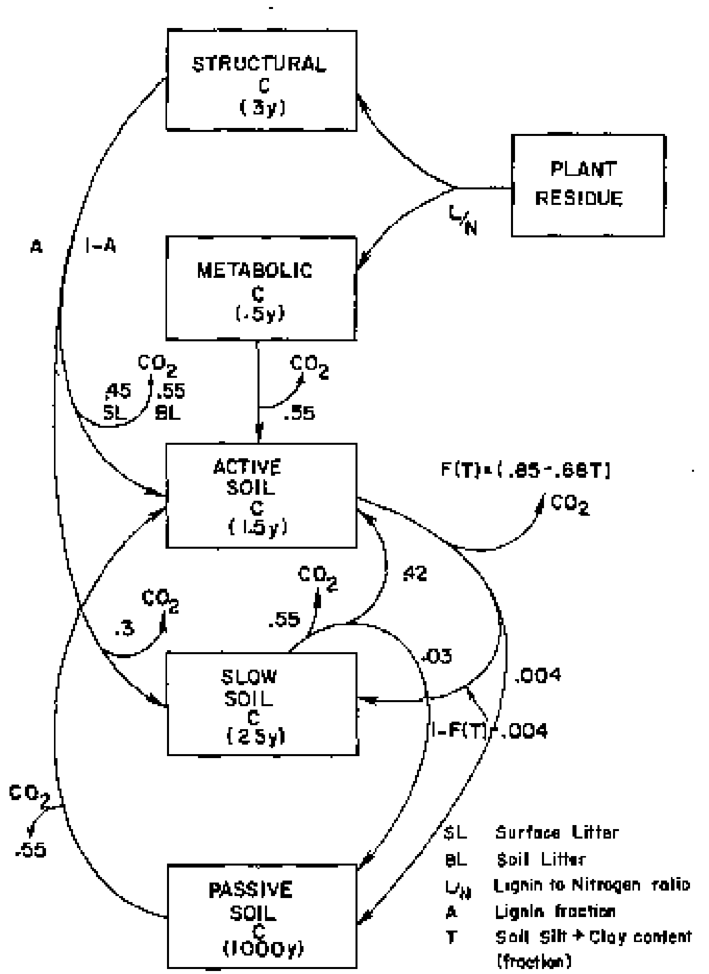


Fig. 1. Flow diagram for the C flows in the Century model.

Parton et al. (1987)

Mathematical implementation

System dynamics

Mathematical manipulations

Steady-state analytical expression

Lardy et al. (2011)
C is the vector of carbon stocks.
The system dynamics is represented as:

$$C'(t) = \rho_t A_t C_t + B_t \qquad (1)$$

with: $$C'(t) = \frac{C_{t+1} - C_t}{dt} \qquad (2)$$

ρ_t represents the temperature and water stresses.
A_t is the matrix of the maximum decomposition rates.
B_t is the vector of inputs.

We rearrange (1) and (2) in:

$$C_{t+1} = (I + \rho_t dt A_t) C_t + B_t dt \qquad (3)$$

and define:

$$D_t = (I + \rho_t dt A_t) \qquad (4)$$

Some mathematical manipulations

$$C_{t+1} = (I + \rho_t dtA_t)C_t + B_t dt \qquad (3)$$

$$D_t = (I + \rho_t dtA_t) \qquad (4)$$

Equation (3) may thus be rewritten:

$$C_{t+1} = D_t C_t + B_t dt \qquad (5)$$

By induction we have:

$$C_t = \sum_{i=t_0}^{t-1} (\prod_{j=i+1}^{t-1} D_j) B_i dt + (\prod_{i=t_0}^{t-1} D_i) C_{t_0} \qquad (6)$$

We introduce the following series:

$$\begin{cases} V_{t_0} = D_{t_0} \\ V_t = D_t V_{t-1} \end{cases} \qquad (7) \qquad \text{and} \qquad \begin{cases} U_{t_0} = B_{t_0} dt \\ U_t = D_t U_{t-1} + B_t dt \end{cases} \qquad (8)$$

$$V_t = D_t V_{t-1} = \prod_{i=t_0}^{t} D_i \qquad (9) \qquad U_t = \sum_{i=t_0}^{t} (\prod_{j=i+1}^{t} D_j) B_i dt \qquad (10)$$

Steady-state analytical expression

$$C_t = \sum_{i=t_0}^{t-1} \left(\prod_{j=i+1}^{t-1} D_j\right) B_i dt + \left(\prod_{i=t_0}^{t-1} D_i\right) C_{t_0} \qquad (6)$$

$$V_t = D_t V_{t-1} = \prod_{i=t_0}^{t} D_i \qquad (9)$$

$$U_t = \sum_{i=t_0}^{t} \left(\prod_{j=i+1}^{t} D_j\right) B_i dt \qquad (10)$$

Thus:

$$C_t = U_{t-1} + V_{t-1} C_{t_0} \qquad (11)$$

At equilibrium:

$$C_t = C_{t_0} = C^* \qquad (12)$$

$$C^* = U_{t-1} + V_{t-1} C^* \qquad (13)$$

$$(I - V_{t-1}) C^* = U_{t-1} \qquad (14)$$

$$\boxed{C^* = (I - V_{t-1})^{-1} U_{t-1}} \qquad (15)$$

Code implementation

Initialization part

Loop part

Final inversion

Initialisation part

constantes_var.f90
global variable spinup_analytic initialized to FALSE

constantes.f90::activate_sub_models
SPINUP_ANALYTIC KEYWORD read

stomate.f90::stomate_init
allocation of variables

IF spinup_analytic

stomate.f90::stomate_initialize
KEYWORDs read: SPINUP_PERIOD, EPS_CARBON

stomate_io.f90::readstart
Specific variables are read from the restart file:
‘Global_years’, ‘nbp_sum’, ‘nbp_flux’, ‘ok_equilibrium’,
‘MatrixV’,’VectorU’,’previous_stock’, ‘current_stock’

Loop part

At each sechiba time step

stomate_io.f90::write_restart

IF (lstep_last) specific variables are written in the restart file.

stomate_litter.f90::littercalc

filling of MatrixA and VectorB with fluxes related to litter pools

stomate_soilcarbon.f90::soilcarbon

filling of MatrixA with fluxes related to soil carbon pools

$A = A + I \ (= D)$

At each stomate time step

lpj_fire.f90::fire

update of the terms of MatrixA related to above ground litter pools using firefrac

stomate.f90::stomate_main

nbp is accumulated.

stomate.f90::stomate_main

$V_t = D_t V_{t-1}$

$U_t = D_t U_{t-1} + B_t dt$

stomate.f90::stomate_main

IF LastTsYear

Increment the years counter (global_years).

IF global_years is a multiple of SPINUP_PERIOD

- $C^* = (I - V_{t-1})^{-1} U_{t-1}$ is computed using the Gauss-Jordan method (gauss_jordan_method.f90::gauss_jordan_method).

- Compute the relative error over the passive carbon pool (sum over all PFTs for each pixel, gauss_jordan_method.f90::error_L1_passive).

- For pixels where relative error <= EPS_CARBON
  ok_equilibrium=TRUE

- Update all pools to new values.

- **IF all pixels at equilibrium: END OF THE ANALYTICAL SPINUP**

ENDIF

ENDIF

libIGCM SPINUP_ANALYTIC configuration

Configuration files

Expected evolution of NBP & CARBON_PASSIVE

```
#==========================================================================
#D-- UserChoices -
DateBegin=1901-01-01
DateEnd=2240-12-31
#
# Forcing data between 1901 and 1910
CyclicBegin=1901
CyclicEnd=1910
#==========================================================================
#D-- Post -
#D- Do we rebuild parallel output, this flag determines
#D- frequency of rebuild submission
RebuildFrequency=NONE → 5Y (if IOIPSL)
…
#==========================================================================
#D-- SRF - SECHIBA
[SRF]
WriteFrequency="1Y"

#==========================================================================
#D-- SRF - STOMATE
[SBG]
WriteFrequency="1Y"
```

orchidee_ol.card

```
[BoundaryFiles]
List=     (${R_IN}/SRF/METEO/CRU-NCEP/v5.3.2/twodeg/cruncep_twodeg_${CyclicYear}.nc, forcing_file.nc)
```

sechiba.card

```
[UserChoices]
# VEGET_UPDATE=0Y: no change in vegetation map. PFTmap should be set only in InitialStateFiles/List.
# VEGET_UPDATE=1Y: change vegetation map every year. PFTmap should be added in BoundaryFiles/List.
VEGET_UPDATE=0Y

[InitialStateFiles]
List=    (${R_IN}/SRF/routing.nc, .),     \
       (${R_IN}/SRF/soils_param.nc, .), \
       (${R_IN}/SRF/soils_param_usdatop.nc, .), \
       (${R_IN}/SRF/cartepente2d_15min.nc, .), \
       (${R_IN}/SRF/floodplains.nc, .), \
       (${R_IN}/SRF/reftemp.nc, .), \
       (${R_IN}/SRF/albedo/alb_bg_modisopt_2D.nc, alb_bg.nc) , \
       (${R_IN}/SRF/PFTMAPS/CMIP6/ESA-LUH2/historical/v1.2/withoutNoBio/13PFTmap_1860_ESA
_LUH2v2h_withoutNoBio_v1.2.nc, PFTmap.nc)
```

COMP (continued)

stomate.card

```
[UserChoices]
# SPINUP_ANALYTIC=y/n : Activate the spinup analytic option to solve the carbon in soil balance
SPINUP_ANALYTIC=y

[InitialStateFiles]
List=   ()

[BoundaryFiles]
List=          ()
ListNonDel= ()

[SmoothFiles]
List=          ()

[ParametersFiles]
List=   (${SUBMIT_DIR}/PARAM/run.def, .)

[RestartFiles]
# List restart that have to be saved/restored each loop (file out, saved, and in) :
List=          (stomate_rest_out.nc, stomate_rest.nc, stomate_rest_in.nc)
```

# SECHIBA history output level (0..10)
# default = 5
SECHIBA_HISTLEVEL = 4

# SECHIBA history output level (0..10)
# default = 5
SECHIBA_HISTLEVEL = 5

# Write frequency in days or -1 for monthly output in stomate_ipcc_history.nc
# default = 0.
STOMATE_IPCC_HIST_DT = _AUTO_

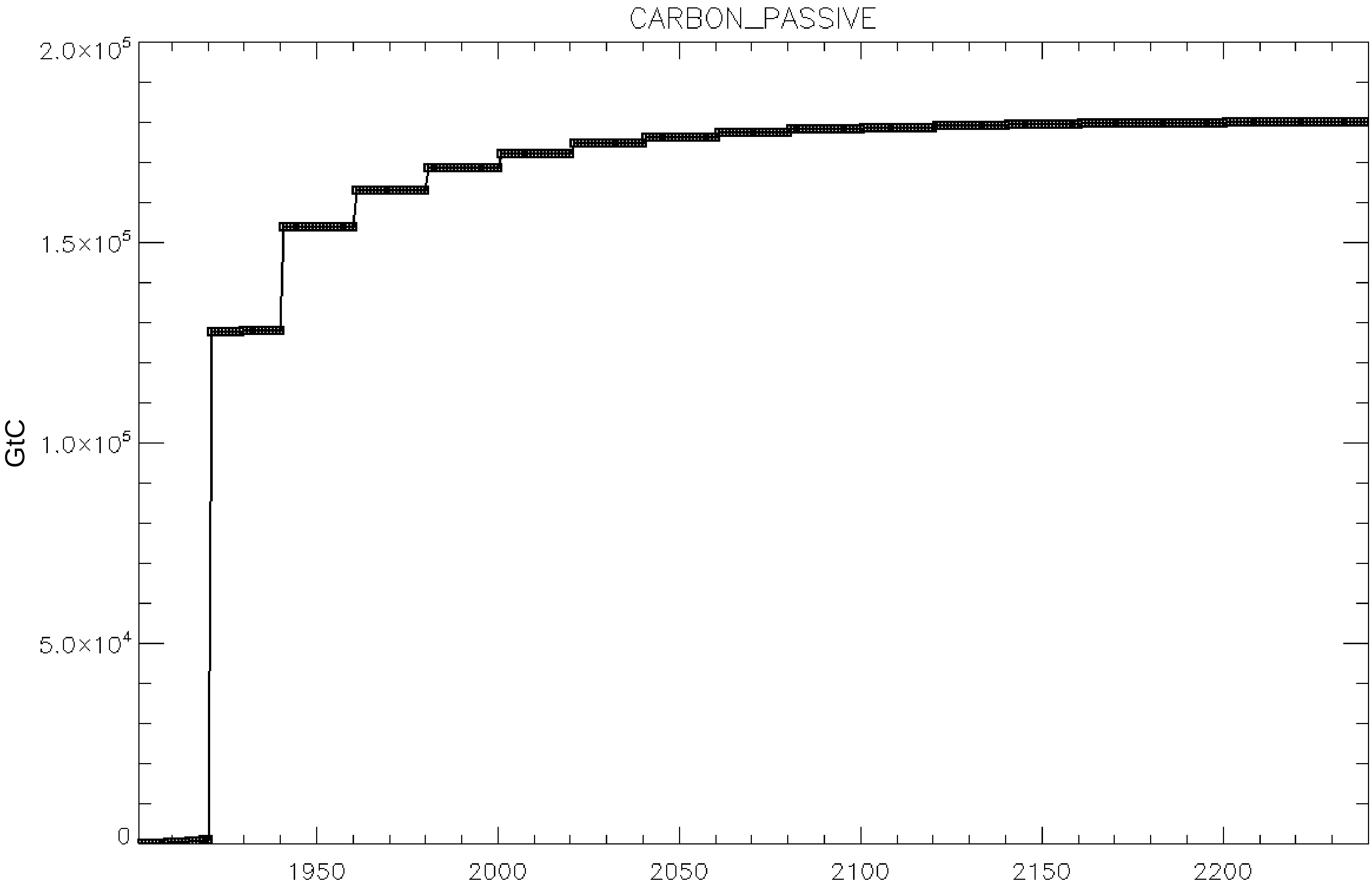
# ATM_CO2=287.14 : Year 1860 specified for TRENDY2 spinup
ATM_CO2 = _AUTO_: DEFAULT = 287.14

# Deactivate fire
FIRE_DISABLE=y

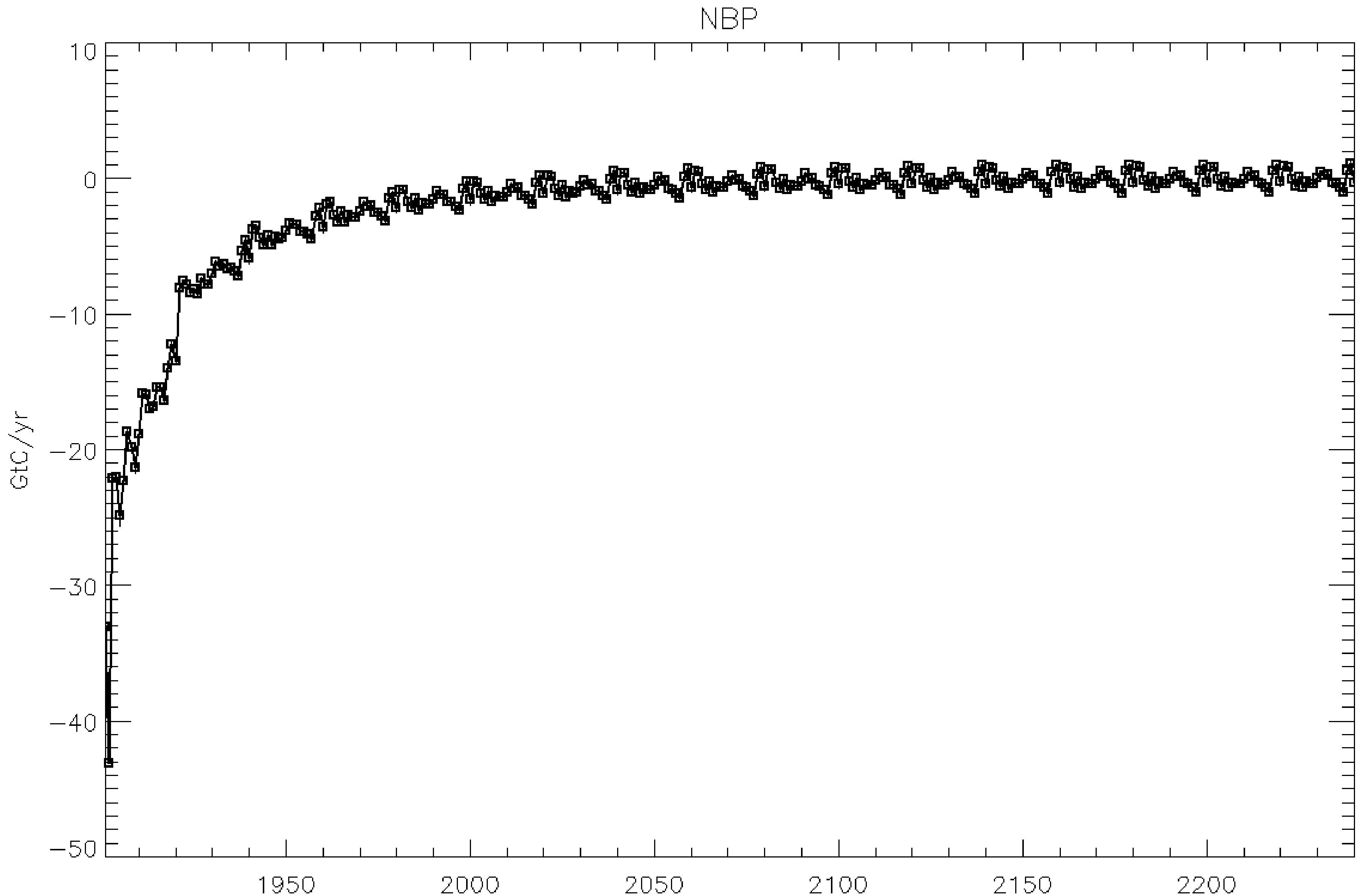
# EPS_CARBON ([%]   ) : Allowed error on carbon stock {SPINUP_ANALYTIC}
EPS_CARBON = 0.01

# SPINUP_PERIOD ([years]   ) : Period to calculate equilibrium during spinup analytic {SPINUP_ANALYTIC}
SPINUP_PERIOD = -1

CARBON_PASSIVE time-series global scale



NBP time-series global scale



Useful references / link

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Work of D. Solyga:
http://forge.ipsl.jussieu.fr/orchidee/wiki/DevelopmentActivities/AccelerationSpinup