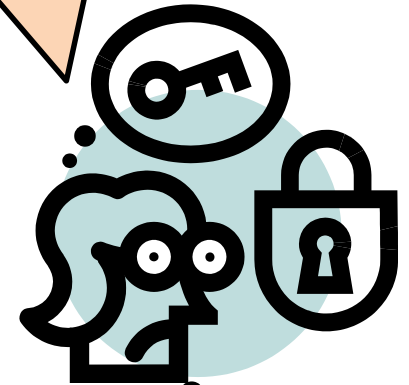


Introduction to the carbon spinup – Analytical solution

What the hell
is this?



Do I need it?



**CARBON
SPINUP**

How can I
do?



Carbon Spinup – Analytical solution

Scientific issue

Mathematical solution

ORCHIDEE

Code implementation

libIGCM configuration

Implementation done by D. Solyga, N. Vuichard and J. Ghattas



Scientific issue

Use of Terrestrial Biosphere Models?

Steady state

Carbon Spinup

Dynamics of carbon pools

Case with two carbon pools

In ORCHIDEE

CENTURY model



What is the use of Terrestrial Biosphere Models?

- Test our understanding of processes.
- Lead experiments, for example introduction of disturbances such as:
 - Climate Change
 - Increasing atmospheric CO₂ concentration
 - Land Use Change
 - ...
- ➔ A stable initialization state is required to study impacts.



- Without disturbances, the carbon cycle in terrestrial ecosystems and in TBMs reaches an equilibrium, with all variables at a steady state (mean stability over a forcing period).
- This steady state equilibrium is usually used as the initialization state of experiments with TBM (especially in MIP).
- This ideal state is useful when you don't know the real history (plantation date, fires, LUC, ...) so always with regional/global simulations.
- This equilibrium depends on climate, vegetation type, soil.



This equilibrium is usually reached by running the carbon model **several thousands of years** to bring all carbon pools at equilibrium.

This operation is named as the **spinup** of the model.

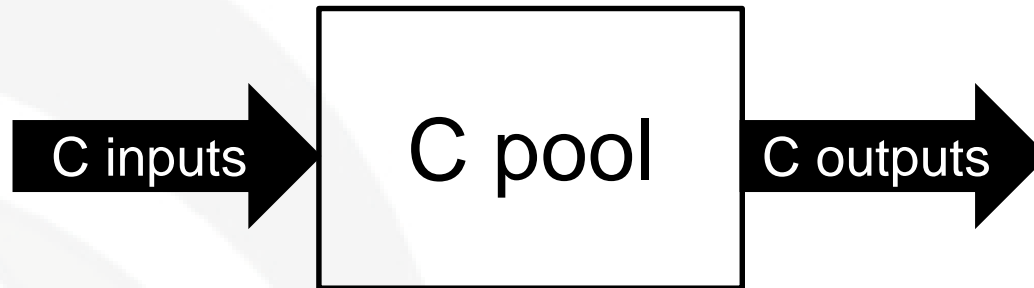
The computational cost is quite heavy as compared to the experiment itself (hundreds of years).

- ⇒ We need to optimise the spinup.
- ⇒ Analytical expression of the carbon stocks at the equilibrium state



Dynamics of carbon pools

Carbon processes in terrestrial ecosystems can be represented by **linear first-order differential equations**.



b influx

$$\frac{dC}{dt} = -aC + b$$

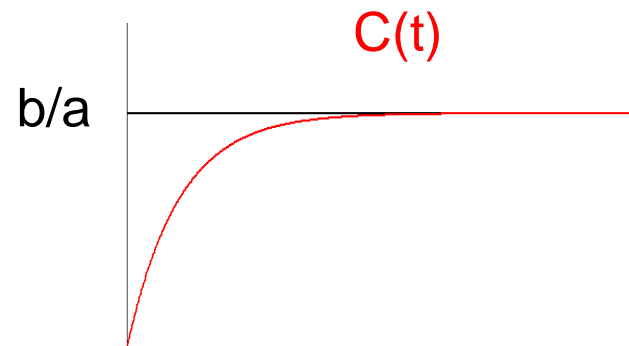
a decomposition rate ($a > 0$)

$$a = A\rho_T\rho_W$$

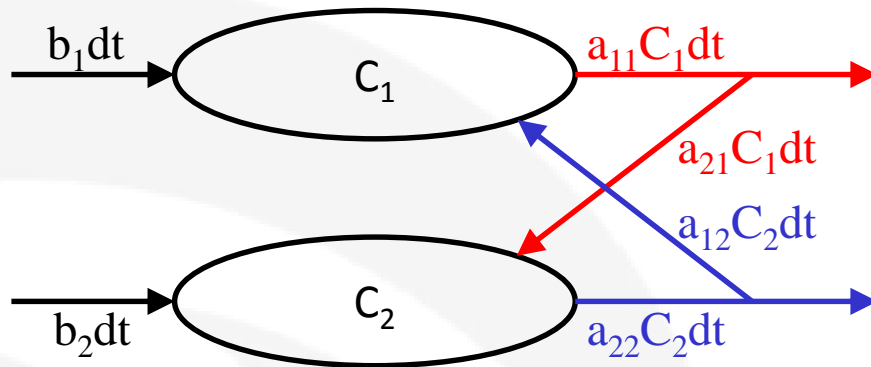
$\tau = 1/a$ residence time

Simple case: one pool, a and b constants, $C(0)=0$

$$\Rightarrow C(t) = \frac{b}{a}(1 - e^{-at})$$



Case with two carbon pools



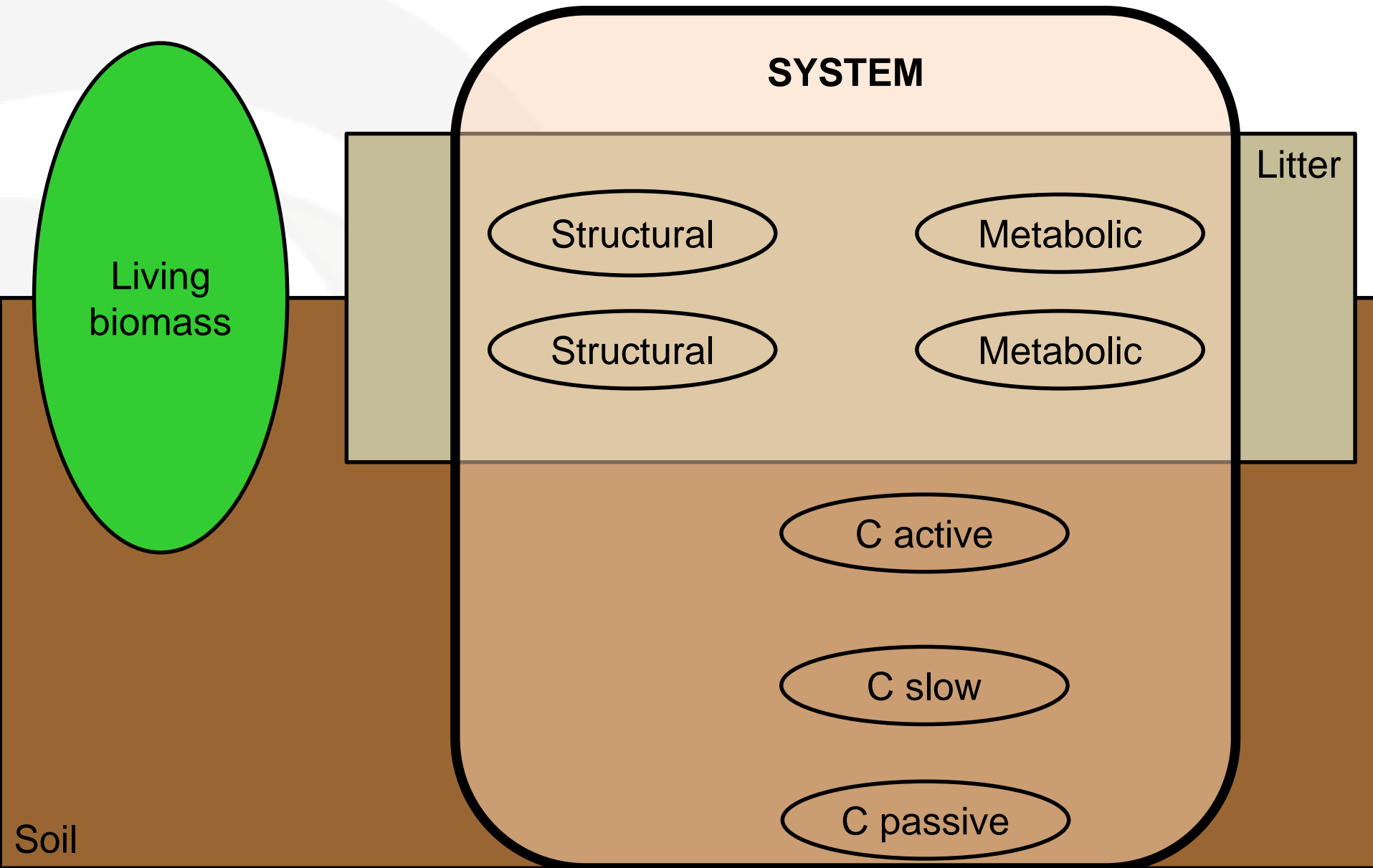
$$\frac{dC_1}{dt} = -a_{11} C_1 + a_{12} C_2 + b_1$$

$$\frac{dC_2}{dt} = a_{21} C_1 + a_{22} C_2 + b_2$$

$$\frac{d}{dt} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -a_{11} & a_{12} \\ a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\frac{dC}{dt} = AC + B$$





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SOIL SCI. SOC. AM. .

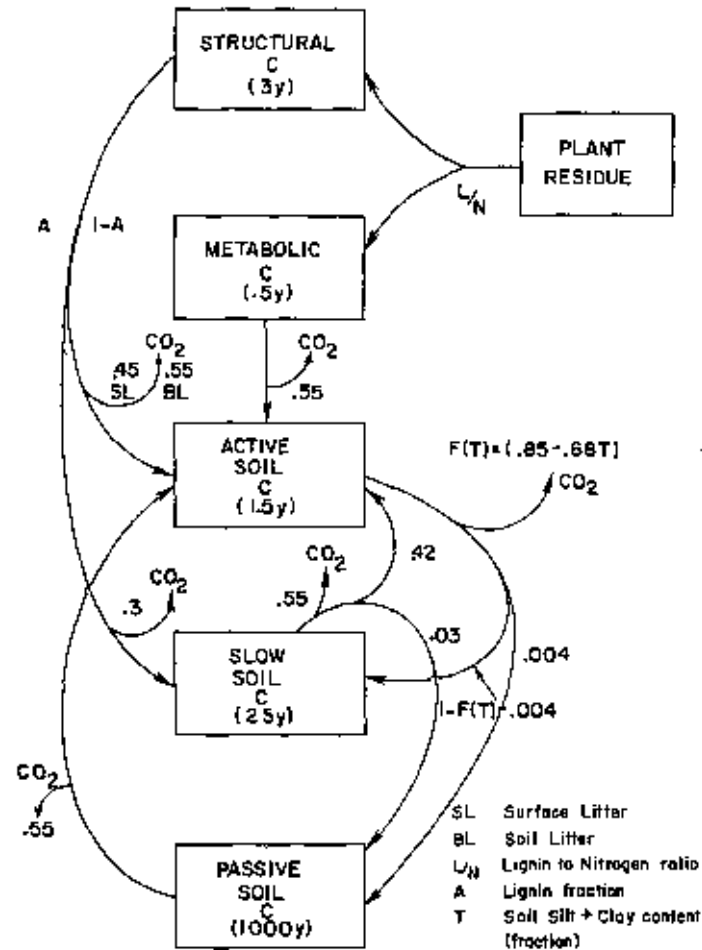


Fig. 1. Flow diagram for the C flows in the Century model.

Parton et al. (1987)



Mathematical implementation

System dynamics

Mathematical manipulations

Steady-state analytical expression



Lardy et al. (2011)

C is the vector of carbon stocks.

The system dynamics is represented as:

$$C'(t) = \rho_t A_t C_t + B_t \quad (1)$$

with:
$$C'(t) = \frac{C_{t+1} - C_t}{dt} \quad (2)$$

ρ_t represents the temperature and water stresses.

A_t is the matrix of the maximum decomposition rates.

B_t is the vector of inputs.

We rearrange (1) and (2) in:

$$C_{t+1} = (I + \rho_t dt A_t) C_t + B_t dt \quad (3)$$

and define:

$$D_t = (I + \rho_t dt A_t) \quad (4)$$



Some mathematical manipulations

$$C_{t+1} = (I + \rho_t dt A_t) C_t + B_t dt \quad (3)$$

$$D_t = (I + \rho_t dt A_t) \quad (4)$$

Equation (3) may thus be rewritten:

$$C_{t+1} = D_t C_t + B_t dt \quad (5)$$

By induction we have:

$$C_t = \sum_{i=t_0}^{t-1} \left(\prod_{j=i+1}^{t-1} D_j \right) B_i dt + \left(\prod_{i=t_0}^{t-1} D_i \right) C_{t_0} \quad (6)$$

We introduce the following series:

$$\begin{cases} V_{t_0} = D_{t_0} \\ V_t = D_t V_{t-1} \end{cases} \quad (7)$$

$$\text{and } \begin{cases} U_{t_0} = B_{t_0} dt \\ U_t = D_t U_{t-1} + B_t dt \end{cases} \quad (8)$$

$$V_t = D_t V_{t-1} = \prod_{i=t_0}^t D_i \quad (9)$$

$$U_t = \sum_{i=t_0}^t \left(\prod_{j=i+1}^t D_j \right) B_i dt \quad (10)$$



Steady-state analytical expression

$$C_t = \sum_{i=t_0}^{t-1} \left(\prod_{j=i+1}^{t-1} D_j \right) B_i dt + \left(\prod_{i=t_0}^{t-1} D_i \right) C_{t_0} \quad (6)$$

$$V_t = D_t V_{t-1} = \prod_{i=t_0}^t D_i \quad (9)$$

$$U_t = \sum_{i=t_0}^t \left(\prod_{j=i+1}^t D_j \right) B_i dt \quad (10)$$

Thus:

$$C_t = U_{t-1} + V_{t-1} C_{t_0} \quad (11)$$

At equilibrium:

$$C_t = C_{t_0} = C^* \quad (12)$$

$$C^* = U_{t-1} + V_{t-1} C^* \quad (13)$$

$$(I - V_{t-1}) C^* = U_{t-1} \quad (14)$$

$$C^* = (I - V_{t-1})^{-1} U_{t-1} \quad (15)$$



Initialization part

Loop part

Final inversion



constantes_var.f90

global variable `spinup_analytic` initialized to FALSE

constantes.f90::activate_sub_models

`SPINUP_ANALYTIC` KEYWORD read

stomate.f90::stomate_init

allocation of variables

IF `spinup_analytic`

stomate.f90::stomate_initialize

KEYWORDS read: `SPINUP_PERIOD`, `EPS_CARBON`

stomate_io.f90::readstart

Specific variables are read from the restart file:

'Global_years', 'nbp_sum', 'nbp_flux', 'ok_equilibrium',
'MatrixV', 'VectorU', 'previous_stock', 'current_stock'



At each sechiba time step

stomate_io.f90::write_restart

IF (lstep_last) specific variables are written in the restart file.

stomate_litter.f90::littercalc

filling of MatrixA and VectorB with fluxes related to litter pools

stomate_soilcarbon.f90::soilcarbon

filling of MatrixA with fluxes related to soil carbon pools

$A = A + I (=D)$

At each stomate time step

lpj_fire.f90::fire

update of the terms of MatrixA related to above ground litter pools using firefrac

stomate.f90::stomate_main

nbp is accumulated.

stomate.f90::stomate_main

$$V_t = D_t V_{t-1}$$

$$U_t = D_t U_{t-1} + B_t dt$$

stomate.f90::stomate_main

IF EndOf Year

Increment the years counter (global_years).

IF global_years is a multiple of SPINUP_PERIOD

- $C^* = (I - V_{t-1})^{-1} U_{t-1}$ is computed using the Gauss-Jordan method (`gauss_jordan_method.f90::gauss_jordan_method`).
- Compute the relative error over the passive carbon pool (sum over all PFTs for each pixel, `gauss_jordan_method.f90::error_L1_passive`).
- For pixels where relative error < EPS_CARBON
ok_equilibrium=TRUE
- Update all pools to new values.

• IF all pixels at equilibrium: END OF THE ANALYTICAL SPINUP

ENDIF

ENDIF



Configuration files

Expected evolution of
NBP & CARBON_PASSIVE



```
#=====
#D-- UserChoices -
DateBegin=1901-01-01
DateEnd=2240-12-31
#
# Forcing data between 1901 and 1910
CyclicBegin=1901
CyclicEnd=1910
#=====
#D-- Post -
#D- Do we rebuild parallel output, this flag determines
#D- frequency of rebuild submission
RebuildFrequency=NONE → 5Y (if IOIPSL)
...
#=====
#D-- SRF - SECHIBA
[SRF]
WriteFrequency="1Y"
#=====
#D-- SRF - STOMATE
[SBG]
WriteFrequency="1Y"
```



orchidee_ol.card

[BoundaryFiles]

```
List=      (${R_IN}/SRF/METEO/CRU-NCEP/v5.3.2/twodeg/cruncep_twodeg_${CyclicYear}.nc, forcing_file.nc)
```

sechiba.card

[UserChoices]

```
# VEGET_UPDATE=0Y no change in vegetation map. PFTmap should be set only in InitialStateFiles/List.
# VEGET_UPDATE=1Y change vegetation map every year. PFTmap should be added in BoundaryFiles/List and
removed from InitialStateFiles/List.
VEGET_UPDATE=0Y
```

[InitialStateFiles]

```
List=      (${R_IN}/SRF/routing.nc, .), \
            (${R_IN}/SRF/soils_param.nc, .), \
            (${R_IN}/SRF/soils_param_usdatop.nc, .), \
            (${R_IN}/SRF/cartepente2d_15min.nc, .), \
            (${R_IN}/SRF/floodplains.nc, .), \
            (${R_IN}/SRF/reftemp.nc, .), \
            (${R_IN}/SRF/albedo/alb_bg_modisopt_2D.nc, alb_bg.nc) , \
            (${R_IN}/SRF/PFTmap_1850to2005_AR5_LUHa.rc2/PFTmap_IPCC_1860.nc, PFTmap.nc)
```



stomate.card

[UserChoices]

SPINUP_ANALYTIC=y/n : Activate the spinup analytic option to solve the carbon in soil balance

SPINUP_ANALYTIC=y

[InitialStateFiles]

List= ()

[BoundaryFiles]

List= ()

ListNonDel= ()

[SmoothFiles]

List= ()

[ParametersFiles]

List= (\${SUBMIT_DIR}/PARAM/run.def, .)

[RestartFiles]

List restart that have to be saved/restored each loop (file out, saved, and in) :

List= (stomate_rest_out.nc, stomate_rest.nc, stomate_rest_in.nc)



```
# SECHIBA history output level (0..10)
```

```
# default = 5
```

```
SECHIBA_HISTLEVEL = 11
```

```
# SECHIBA history output level (0..10)
```

```
# default = 5
```

```
SECHIBA_HISTLEVEL = 11
```

```
# Write frequency in days or -1 for monthly output in stomate_ipcc_history.nc
```

```
# default = 0.
```

```
STOMATE_IPCC_HIST_DT = _AUTO_
```

```
# ATM_CO2=287.14 : Year 1860 specified for TRENDY2 spinup
```

```
ATM_CO2 = _AUTO_ : DEFAULT = 287.14
```

```
# Deactivate fire
```

```
FIRE_DISABLE=y
```

```
# EPS_CARBON ([%] ) : Allowed error on carbon stock {SPINUP_ANALYTIC}
```

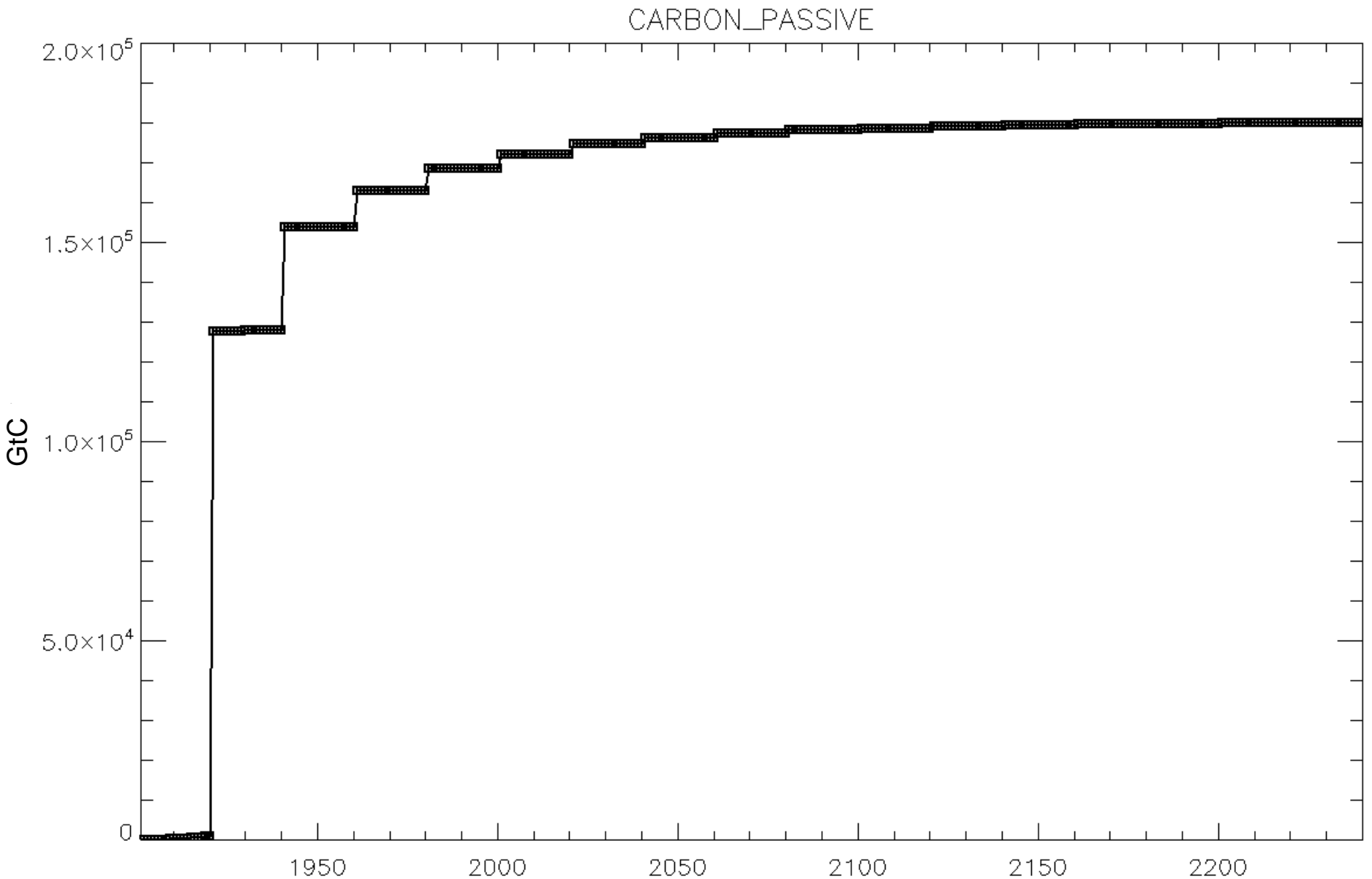
```
EPS_CARBON = 0.01
```

```
# SPINUP_PERIOD ([years] ) : Period to calculate equilibrium during spinup analytic {SPINUP_ANALYTIC}
```

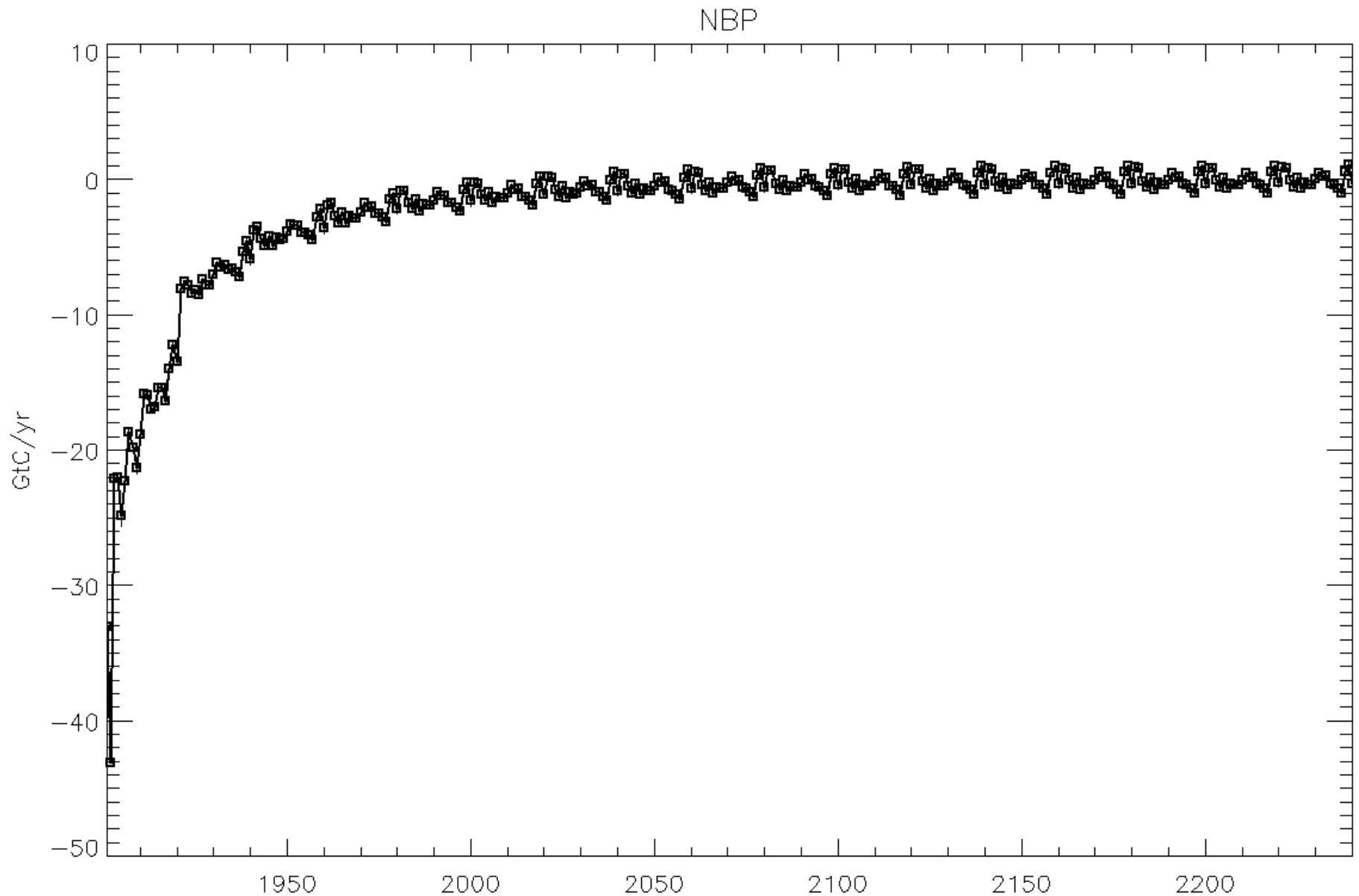
```
SPINUP_PERIOD = -1
```



CARBON_PASSIVE time-series global scale



NBP time-series global scale



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Work of D. Solyga:

<http://forge.ipsl.jussieu.fr/orchidee/wiki/DevelopmentActivities/AccelerationSpinup>

