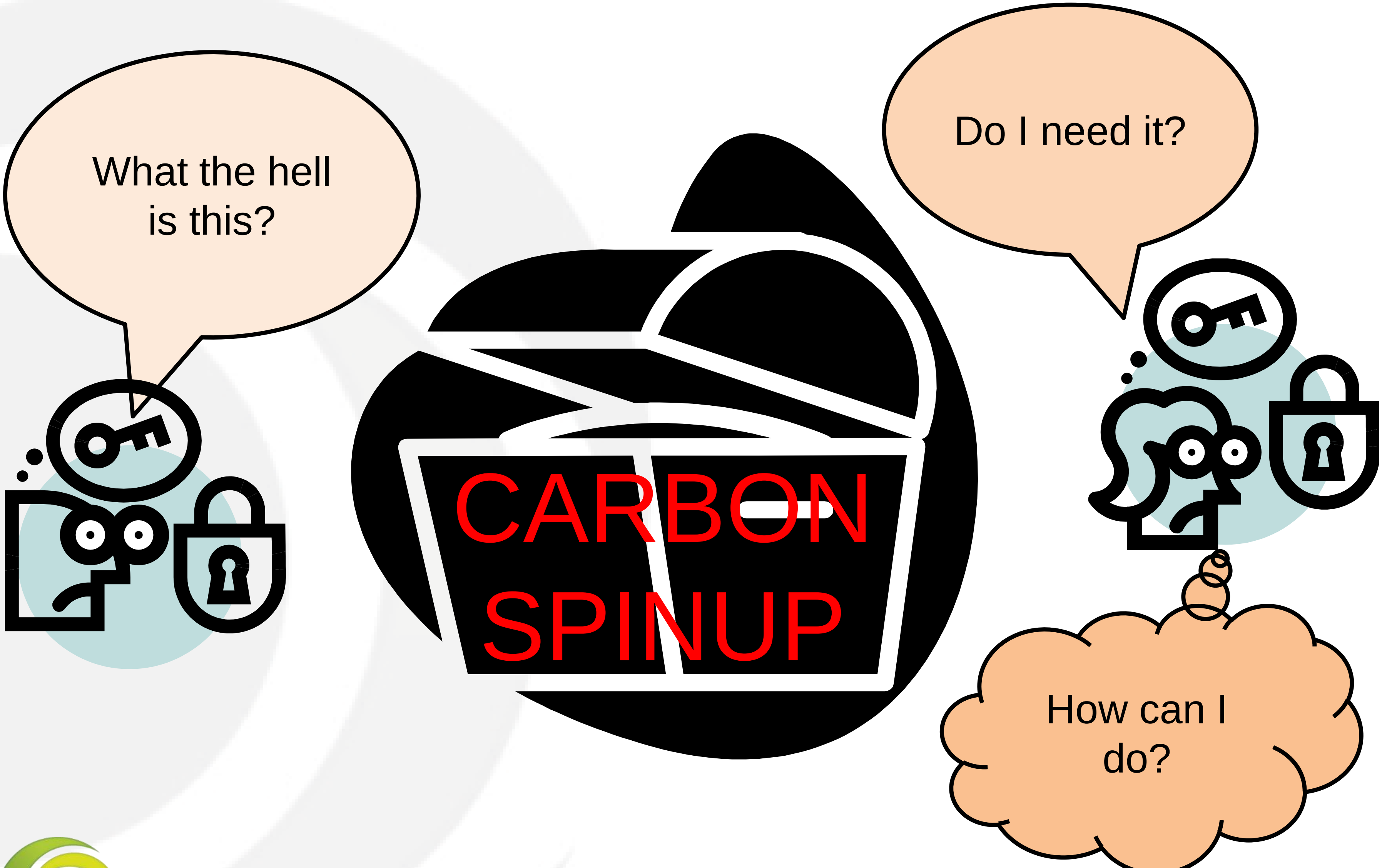
Introduction to the carbon spinup – analytical solution



Carbon Spinup – Analytical solution

- Scientific issue
- Mathematical solution
- ORCHIDEE
- Code implementation
- libIGCM configuration

Implementation done by D. Solyga, N. Vuichard and J. Ghattas

Scientific issue

- Use of Terrestrial Biosphere Models?
- Steady state
- Carbon Spinup
- Dynamics of carbon pools
- Case with two carbon pools
- In ORCHIDEE
- CENTURY model

What is the use of Terrestrial Biosphere Models?

→ Test our understanding of processes.

→ Lead experiments, for example introduction of disturbances such as:

- Climate Change
- Increasing atmospheric CO_2 concentration
- Land Use Change
- …

➲ A stable initialisation state is required to study impacts.

- Without disturbances, the carbon cycle in terrestrial ecosystems and in TBMs reaches an equilibrium, with all variables at a steady state (mean stability over a forcing period).
- This steady state equilibrium is usually used as the initialization state of experiments with TBM (especially in MIP).
- This ideal state is useful when you don't know the real history (plantation date, fires, LUC, …) so always with regional/global simulations.
- This equilibrium depends on climate, vegetation type, soil.

Carbon Spinup

This equilibrium is usually reached by running the carbon model **several thousands of years** to bring all carbon pools at equilibrium.

This operation is named as the **spinup** of the model.

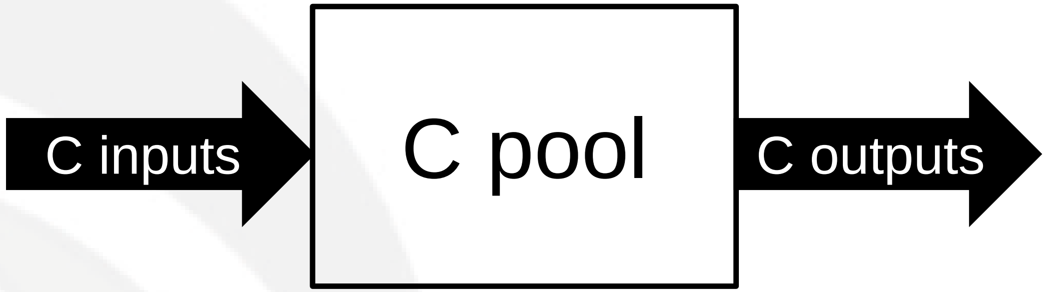
The computational cost is quite heavy as compared to the experiment itself (hundreds of years).

➲ We need to optimise the spinup.

➲ Analytical expression of the carbon stocks at the equilibrium state

Dynamics of carbon pools

Carbon processes in terrestrial ecosystems can be represented by **linear first-order differential equations**.



b influx

$$\frac{dC}{dt} = -aC + b$$

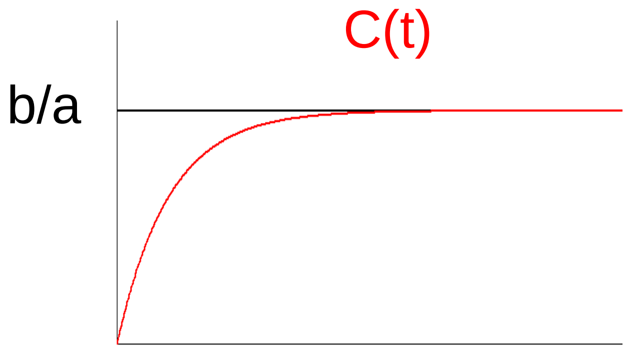
a decomposition rate (a>0)

$a = A\rho_T\rho_W$

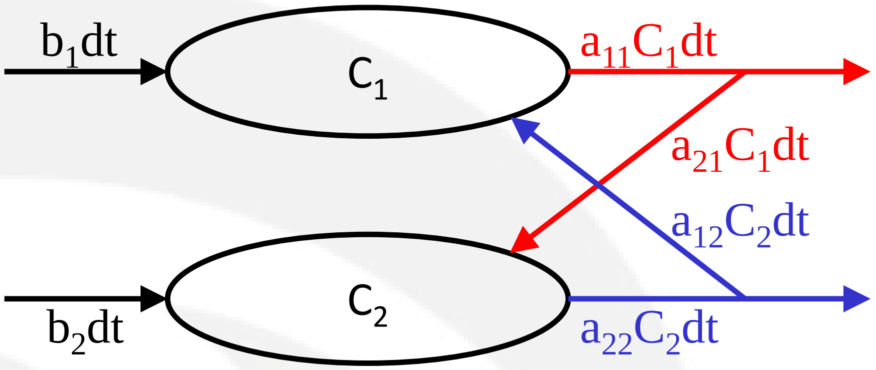
τ = 1/a residence time

Simple case: one pool, a and b constants, C(0)=0

$$\Rightarrow \quad C(t) = \frac{b}{a}(1 - e^{-at})$$



Case with two carbon pools



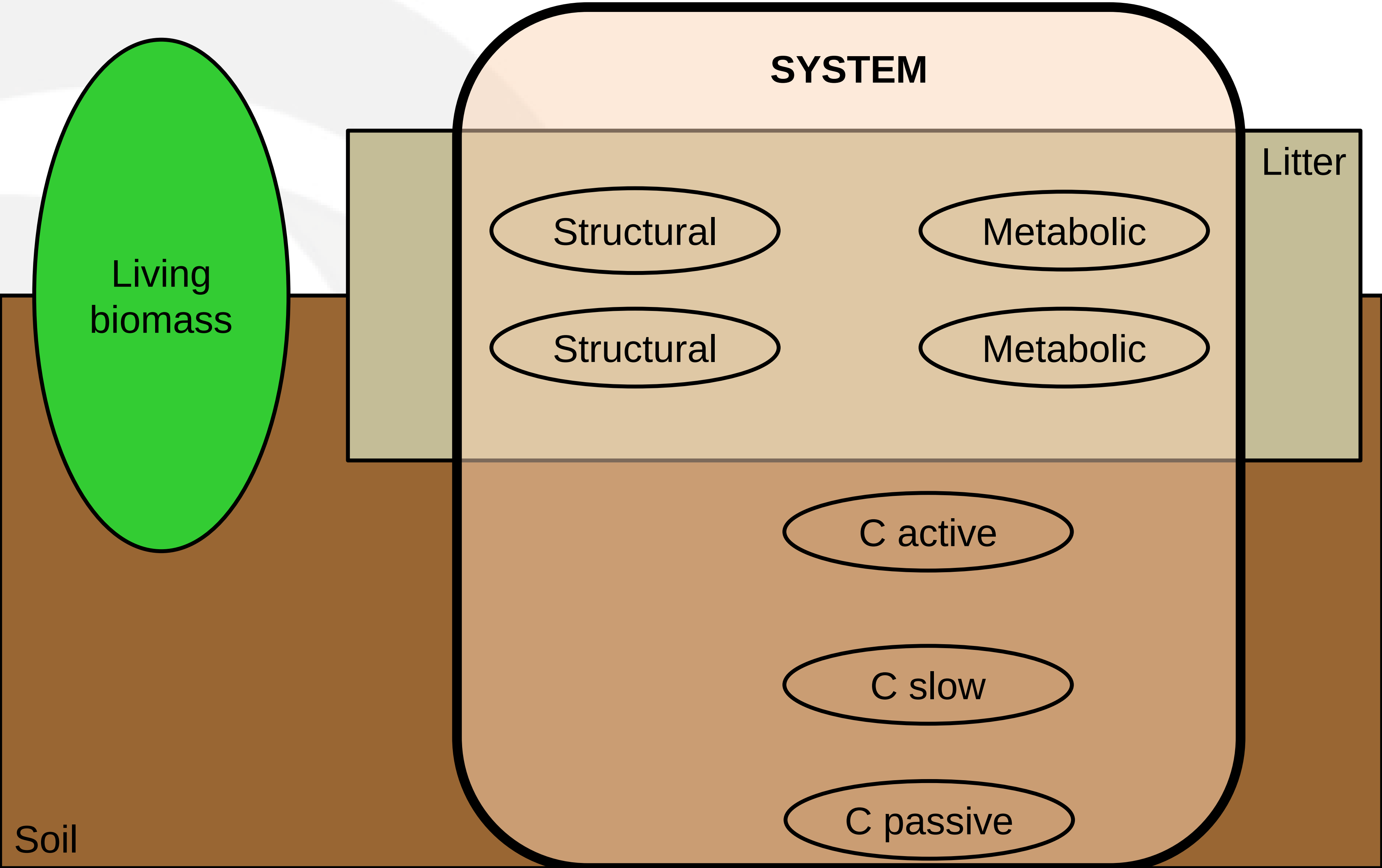
$$\frac{dC_1}{dt} = -a_{11}C_1 + a_{12}C_2 + b_1$$

$$\frac{dC_2}{dt} = a_{21}C_1 + a_{22}C_2 + b_2$$

$$\frac{d}{dt}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -a_{11} & a_{12} \\ a_{21} & -a_{22} \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\frac{dC}{dt} = AC + B$$

In ORCHIDEE


SYSTEM
Litter
Structural
Metabolic
Structural
Metabolic
Living biomass
C active
C slow
C passive
Soil

CENTURY model

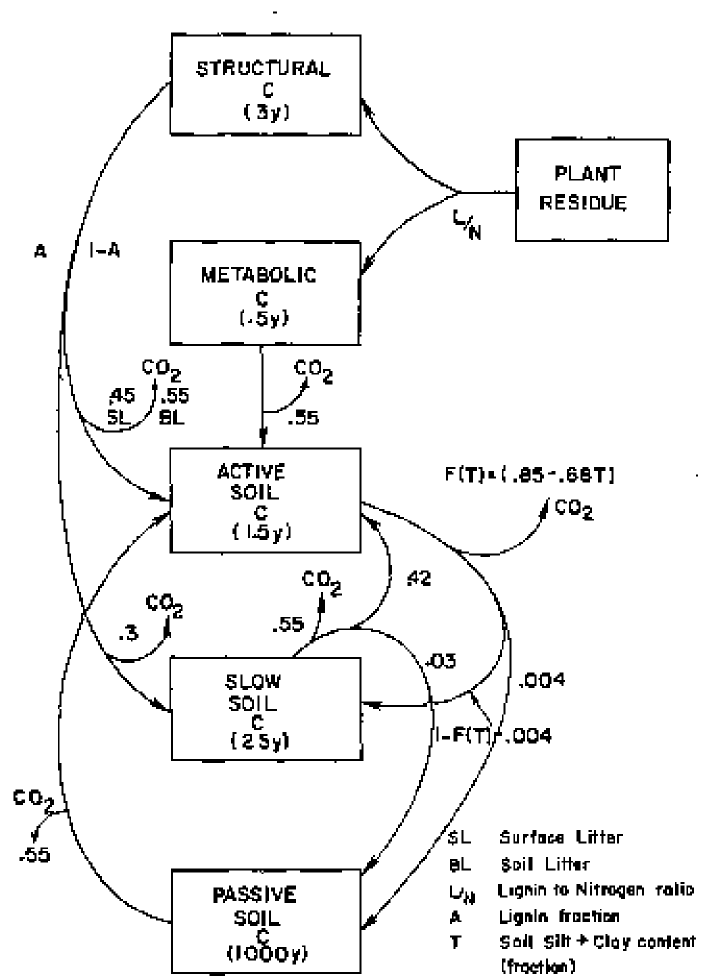


Fig. 1. Flow diagram for the C flows in the Century model.

Parton et al. (1987)

Mathematical implementation

System dynamics

Mathematical manipulations

Steady-state analytical expression

Lardy et al. (2011)
C is the vector of carbon stocks.
The system dynamics is represented as:

$$C'(t) = \rho_t A_t C_t + B_t \quad (1)$$

with:

$$C'(t) = \frac{C_{t+1} - C_t}{dt} \quad (2)$$

ρ_t represents the temperature and water stresses
A_t is the matrix of the maximum decomposition rates
B_t is the vector of inputs

We rearrange (1) and (2) in:

$$C_{t+1} = (I + \rho_t dt A_t) C_t + B_t dt \quad (3)$$

and define:

$$D_t = (I + \rho_t dt A_t) \quad (4)$$

Some mathematical manipulations

$$C_{t+1} = (I + \rho_t dt A_t) C_t + B_t dt \quad (3)$$

$$D_t = (I + \rho_t dt A_t) \quad (4)$$

Equation (3) may thus be rewritten:

$$C_{t+1} = D_t C_t + B_t dt \quad (5)$$

By induction we have:

$$C_t = \sum_{i=t_0}^{t-1} \left(\prod_{j=i+1}^{t-1} D_j \right) B_i dt + \left(\prod_{i=t_0}^{t-1} D_i \right) C_{t_0} \quad (6)$$

We introduce the following series:

$$\begin{cases} V_{t_0} = D_{t_0} \\ V_t = D_t V_{t-1} \end{cases} \quad (7) \qquad \text{and} \qquad \begin{cases} U_{t_0} = B_{t_0} dt \\ U_t = D_t U_{t-1} + B_t dt \end{cases} \quad (8)$$

$$V_t = D_t V_{t-1} = \prod_{i=t_0}^{t} D_i \quad (9) \qquad U_t = \sum_{i=t_0}^{t} \left(\prod_{j=i+1}^{t} D_j \right) B_i dt \quad (10)$$

Steady-state analytical expression

$$C_t = \sum_{i=t_0}^{t-1} \left(\prod_{j=i+1}^{t-1} D_j \right) B_i dt + \left(\prod_{i=t_0}^{t-1} D_i \right) C_{t_0} \quad (6)$$

$$V_t = D_t V_{t-1} = \prod_{i=t_0}^{t} D_i \quad (9)$$

$$U_t = \sum_{i=t_0}^{t} \left(\prod_{j=i+1}^{t} D_j \right) B_i dt \quad (10)$$

Thus:

$$C_t = U_{t-1} + V_{t-1} C_{t_0} \quad (11)$$

At equilibrium:

$$C_t = C_{t_0} = C^* \quad (12)$$

$$C^* = U_{t-1} + V_{t-1} C^* \quad (13)$$

$$(I - V_{t-1}) C^* = U_{t-1} \quad (14)$$

$$C^* = (I - V_{t-1})^{-1} U_{t-1} \quad (15)$$

Code implementation

Initialization part

Loop part

Final inversion

constantes_var.f90
global variable spinup_analytic initialized to FALSE

constantes.f90::activate_sub_models
SPINUP_ANALYTIC KEYWORD read

stomate.f90::stomate_init
allocation of variables

IF spinup_analytic

stomate.f90::stomate_main
KEYWORDs read: SPINUP_PERIOD, EPS_CARBON

stomate_io.f90::readstart
Specific variables are read from the restart file:
'Global_years', 'nbp_sum', 'nbp_flux', 'ok_equilibrium',
'MatrixV','VectorU','previous_stock', 'current_stock'

Loop part

At each sechiba time step

stomate_io.f90::write_restart

IF (last call) specific variables are written in the restart file.

stomate_litter.f90::littercalc

filling of MatrixA and VectorB with fluxes related to litter pools

stomate_soilcarbon.f90::soilcarbon

filling of MatrixA with fluxes related to soil carbon pools

A = A + I (=D)

At each stomate time step

lpj_fire.f90::fire

update of the terms of MatrixA related to above ground litter pools using firefrac

stomate.f90::stomate_main

nbp is accumulated.

stomate.f90::stomate_main

$V_t = D_t V_{t-1}$ $U_t = D_t U_{t-1} + B_t dt$

Final inversion

stomate.f90::stomate_main

IF EndOf Year

Increment the years counter (global_years).

IF global_years is a multiple of SPINUP_PERIOD

- $C^{*} = (I - V_{t-1})^{-1} U_{t-1}$ is computed using the Gauss-Jordan method (gauss_jordan_method.f90::gauss_jordan_method).
- Compute the relative error over the passive carbon pool (sum over all PFTs for each pixel, gauss_jordan_method.f90::error_L1_passive).
- For pixels where relative error < EPS_CARBON ok_equilibrium=TRUE.
- Update all pools to new values.
- **IF all pixels at equilibrium: END OF THE ANALYTICAL SPINUP**

ENDIF

ENDIF

libIGCM SPINUP_ANALYTIC configuration

Configuration files

Expected evolution of NBP & CARBON_PASSIVE

config.card

```
#==========================================================================
#D-- UserChoices -
DateBegin=1901-01-01
DateEnd=2240-12-31
#
# Forcing data between 1901 and 1920
CyclicBegin=1901
CyclicEnd=1920
#==========================================================================
#D-- Post -
#D- Do we rebuild parallel output, this flag determines
#D- frequency of rebuild submission
RebuildFrequency=1Y → 5Y
…
#==========================================================================
#D-- SRF - SECHIBA
[SRF]
WriteFrequency="1Y"

#==========================================================================
#D-- SRF - STOMATE
[SBG]
WriteFrequency="1Y"
```

orchidee_ol.card

```
[BoundaryFiles]
List=       (${R_BC}/OOL/${config_UserChoices_TagName}/CRU-NCEP/v5.2/cruncep_${CyclicYear}.nc, forcing_file.nc)
```

sechiba.card

```
[UserChoices]
# VEGET_UPDATE=0Y no change in PFTmap. PFTmap should be set only in InitialStateFiles/List.
# VEGET_UPDATE=1Y change PFTmap every year in december : PFTmap should be added in BoundaryFiles/List and removed from InitialStateFiles/List.
# This parameter is only used if IMPOSE_VEG=n.
VEGET_UPDATE=0Y

# Set LAND_COVER_CHANGE=y if VEGET_UPDATE > 0
LAND_COVER_CHANGE=n

sechiba_LEVEL=4
[InitialStateFiles]
List=     (${R_INIT}/SRF/${config_UserChoices_TagName}/routing.nc, .),     \
          (${R_INIT}/SRF/${config_UserChoices_TagName}/soils_param.nc, .), \
          (${R_INIT}/SRF/${config_UserChoices_TagName}/cartepente2d_15min.nc, .), \
          (${R_INIT}/SRF/${config_UserChoices_TagName}/floodplains.nc, .), \
          (${R_BC}/SRF/${config_UserChoices_TagName}/PFTmap_1850to2012_TRENDY2.v3/trendy2lu_1860.nc, PFTmap.nc)
```

COMP (continued)

stomate.card

```
[UserChoices]
stomate_LEVEL=5
SPINUP_ANALYTIC=y

[InitialStateFiles]
List=   ()

[BoundaryFiles]
List=           ()
ListNonDel= (${R_BC}/SRF/${config_UserChoices_TagName}/reftemp.nc, .)

[SmoothFiles]
List=           ()

[ParametersFiles]
List=   (${SUBMIT_DIR}/PARAM/run.def, .)

[RestartFiles]
# List restart that have to be saved/restored each loop (file out, saved, and in) :
List=           (stomate_rest_out.nc, stomate_rest.nc, stomate_rest_in.nc)
```

# Deactivate stomate_ipcc_history.nc output file
STOMATE_IPCC_HIST_DT=0

# ATM_CO2=287.14 : Year 1860 specified for TRENDY2 spinup
ATM_CO2=287.14

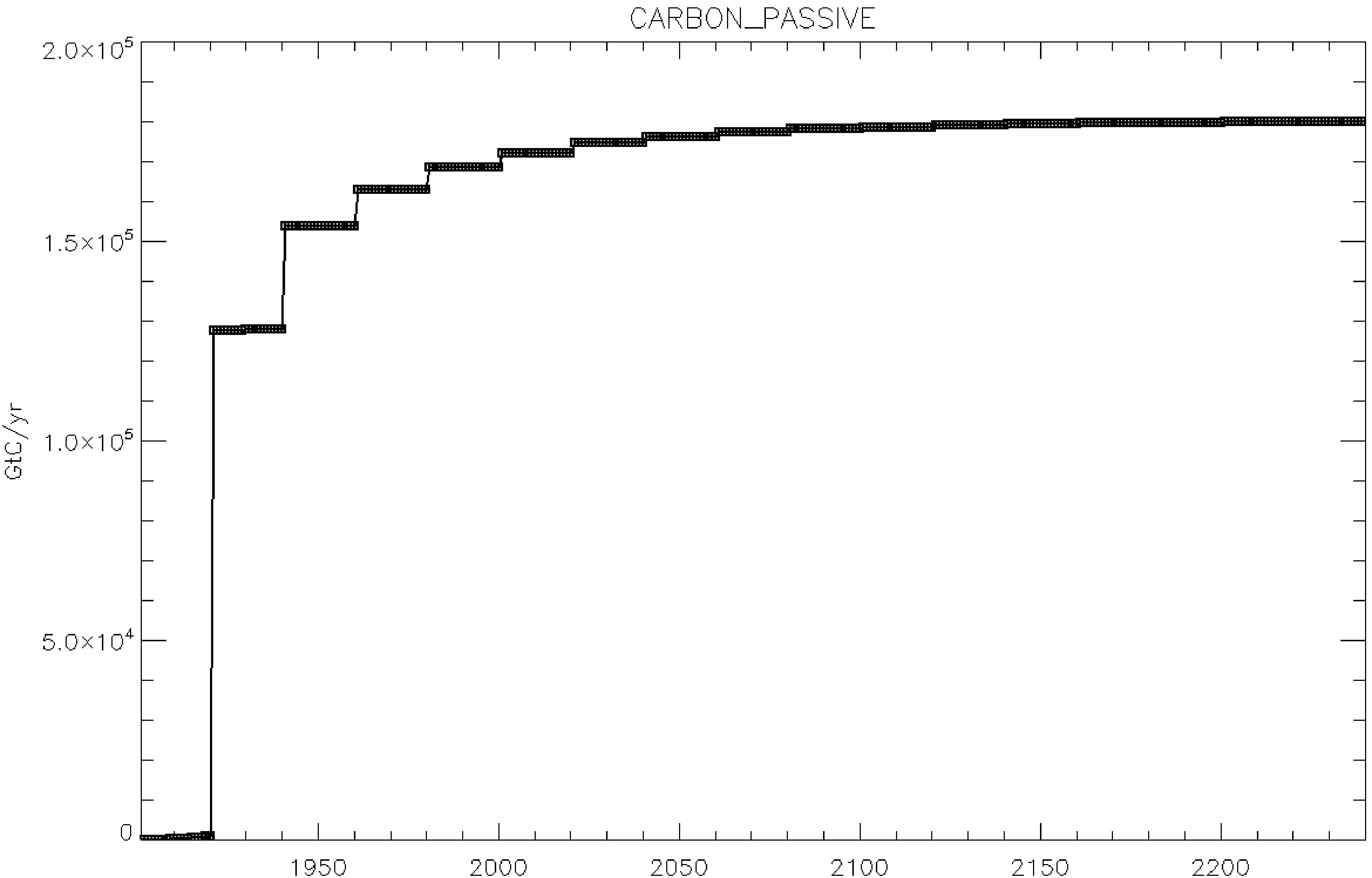
# Deactivate fire
FIRE_DISABLE=y

# Harvest model activated
HARVEST_AGRI=y

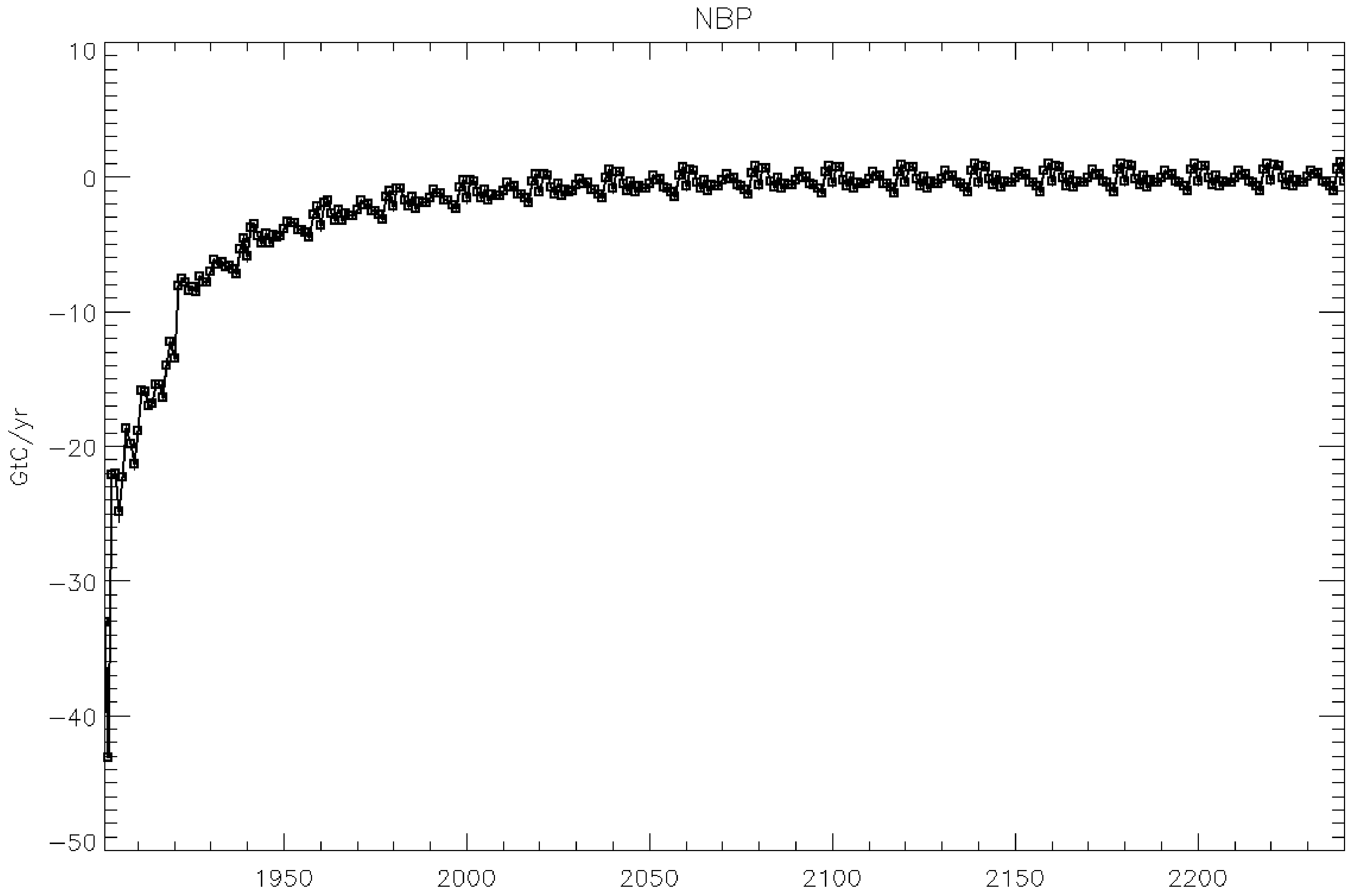
# EPS_CARBON ([%]) : Allowed error on carbon stock
EPS_CARBON = 0.01

SPINUP_PERIOD ([years]) : Period to calculate equilibrium during spinup analytic

CARBON_PASSIVE time-series global scale



NBP time-series global scale



Useful references / link

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Work of D. Solyga: http://forge.ipsl.jussieu.fr/orchidee/wiki/Branches/AccelerationSpinup