Stomatal conductance in JSBACH

New insight from optimization

1. Theoretical Basis

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All of these combinations are special cases of a **generic photosynthesis function**.

The generic photosynthesis function is of the form:

$$A = f\,\frac{c_c - \Gamma_*}{c_c + \gamma}$$

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Basically this reduces to a version of the bi-substrate model, so the solution is very close to Case 1 in the article.

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The generic photosynthesis function is of the form:

$$A = f\,\frac{c_c - \Gamma_*}{c_c + \gamma}$$

When using CAP and Vico's model, we make the following substitutions:

$$f = \phi\,\frac{J(Q)}{4}$$

$$\gamma = \frac{J(Q)\,k_m}{4\,V_{cmax}}$$

$$c_c = c_i$$

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Unfortunately it looks something like this:

$$g_s=\frac{\frac{J(Q)}{\left(\frac{J(Q)k_m}{V_{cmax}}+4\Gamma_*\right)}\left(1-\frac{\Psi_{soil}}{\Psi_0}\right)}{\left(1+\sqrt{\frac{1.6\,D\,J(Q)}{K|\Psi_0|\left(\frac{J(Q)k_m}{V_{cmax}}+4\Gamma_*\right)}}\right)\left(\frac{\frac{1.6\,D\,J(Q)}{K|\Psi_0|\left(\frac{J(Q)k_m}{V_{cmax}}+4\Gamma_*\right)}}{1+\sqrt{\frac{1.6\,D\,J(Q)}{K|\Psi_0|\left(\frac{J(Q)k_m}{V_{cmax}}+4\Gamma_*\right)}}}+\left(1-\frac{1}{1+\sqrt{\frac{1.6\,D\,J(Q)}{K|\Psi_0|\left(\frac{J(Q)k_m}{V_{cmax}}+4\Gamma_*\right)}}}\right)\left(\frac{\frac{C_a-\Gamma_*}{\frac{J(Q)k_m}{4V_{cmax}}+\Gamma_*}}{1+\sqrt{\frac{1.6\,D\,J(Q)}{K|\Psi_0|\left(\frac{J(Q)k_m}{V_{cmax}}+4\Gamma_*\right)}}}+1\right)\right)}$$

1. Theoretical Basis

The exact solution for stomatal conductance is a function of many variables:

$$g_s = g_s\left(C_a, D, K, Q, \Gamma_*, \Psi_{0,} \Psi_{soil}, Farquhar\ parametres\right)$$

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Most of these should be relatively simple in terms of implementation in land surface models. However, with JSBACH, there are a couple of troublemakers.

1. Theoretical Basis

Here you have a “simplified” version:

$$g_s = \frac{J(Q)}{4(\gamma + \Gamma_*)} \frac{x\left(1 - \frac{\Psi_s}{\Psi_0}\right)}{x z_{CAP} + (1 - x)\left(\frac{x(C_a - \Gamma_*)}{\gamma + \Gamma_*} + 1\right)}$$

$$x = \frac{1}{1 + \sqrt{z_{CAP}}}$$

$$z_{CAP} = \frac{1.6\, D\, J(Q)}{4\, K\, |\Psi_0| (\gamma + \Gamma_*)}$$

$$\gamma = \frac{J(Q)\, k_m}{4\, V_{cmax}}$$

2. Photosynthesis in JSBACH

In general, the C3 photosynthesis functions in JSBACH work like the Farquhar model: we have a light-limited regime and a Rubisco-limited regime, and both predict a photosynthetic rate.

The actual predicted rate is the smaller of these two.

2. Photosynthesis in JSBACH

In general, the C3 photosynthesis functions in JSBACH work like the Farquhar model: we have a light-limited regime and a Rubisco-limited regime, and both predict a photosynthetic rate.

The actual predicted rate is the smaller of these two.

(The Vico approach is a sort of an interpolation between the two.)

2. Photosynthesis in JSBACH

Recent versions of JSBACH use the BETHY approach to drought. This means that first an unstressed stomatal conductance is calculated, i.e. a well-watered situation.

Then a second, water-stressed conductance is calculated based on this unstressed one. This is done with a drought factor related to soil water content.

2. Photosynthesis in JSBACH

As it happens, our drought factor is based on soil water *potential,* not soil water *content.*

This requires some adaptation (see for instance Duursma et al. 2008) and a few new parametres:

$$\Psi_{soil} = \Psi_e \left(\frac{\theta}{\theta_{sat}} \right)^{-b}$$

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$$\frac{1}{K} = \frac{1}{K_{xylem}} + \frac{1}{K_{soil}}$$

$$K_{xylem} = constant$$

$$K_{soil} = \frac{R_1}{LAI} \frac{2\pi k_{sat}}{-\log\left(r_{root}\sqrt{(\pi Lv)}\right)} \left(\frac{\theta}{\theta_{sat}}\right)^{2b+3}$$

2. Photosynthesis in JSBACH

There are many new parametres that have to be specified for different plants and environments.

On the other hand, an exact analytical solution may simplify the architecture of the photosynthesis module: for instance, one stomatal conductance can be calculated instead of four.

Also, certain mysterious “empirical constants” now have a meaning.

3. Where Do We Stand?

The theoretical basis seems more or less good to go.

The implementation into the actual code is in an early phase.

There’s still work to be done in mastering the model software, data handling etc.

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- Modify the stomatal conductance function as well as the photosynthesis function.

- Modify both of these as well as the drought function.

3. Where Do We Stand?

The modified models will be run for two 1 km^2 grid cells that represent pine stands in Hyytiälä and Sodankylä.

Later on, we’re planning to do the same for the Aleppo pine stand in Yatir, Israel.

If you have some datasets for this purpose, we’d be eager to use them when the time comes!

3. Where Do We Stand?

These modified model runs will be compared to unmodified JSBACH runs in terms of simulated values for ET, GPP and some internal cost functions included in JSBACH, along the lines of Knauer et al. (2015).

When possible, we’ll also compare the fitted parametres in existing stomatal control functions to the well-determined values predicted by our solution.

4. Issues

Fitted vs. predicted

Our model predicts values for parametres that in existing models are fitted. This is not quite a fair comparison. We’ll compare our predictions with typical fitted values, but **is this enough?**

Another solution could be to treat a portion of the soil-to-root hydraulic conductance or the leaf critical water potential as a fitted parametre and see if it gets realistic values. Or just insert an ad hoc factor somewhere, fit it and hope it comes close to one.

4. Issues

Field values vs. model simplifications

We have parametre values for Hyytiälä that differ from those determined by JSBACH. LAI is a typical example: JSBACH has its own algorithms for determining LAI for any grid cell, but it’s quite far from the one that’s widely in use for Hyytiälä. **Which one to use?**

Also, some of the parametre values (e.g. xylem conductivity) are more like general, realistic estimates than Hyytiälä-specific, measured values.

4. Issues

Soil layers and water

JSBACH currently uses five soil layers for water status analysis (although this can be adjusted if needed). Our model is currently using just one value for soil water potential. That is, our model is a coarse simplification of root water uptake. **Is this a problem?**

4. Issues

Critical water potentials

Our model uses critical leaf water potential as a parametre. JSBACH in turn uses a “permanent wilting point” soil water content, that’s the same for all plants but varies with soil type and can be deduced from literature. A critical soil water potential can be calculated from the PWP soil water content, but I haven’t checked the conversion to leaf water potential, and the interpretation of a PWP is unclear. (Apparently the origin of the PWP value is the equivalent of a soil water potential of -1.5 MPa.) **Is this worrying?**

5. Symbols

Symbol	Variable	Unit
A	photosynthesis rate	mol m^{-2} s^{-1}
c_i	intercellular CO2 concentration	mol mol^{-1}
C_a	ambient CO2 concentration	mol mol^{-1}
D	VPD	mol mol^{-1}
g_s	stomatal conductance	mol m^{-2} s^{-1}
$J(Q)$	electron transport rate (as in Farquhar model)	mol m^{-2} s^{-1}
k_m	Michaelis-Menten coefficient for Rubisco-limited Farquhar model	mol mol^{-1}

5. Symbols

Symbol	Variable	Unit
K	xylem hydraulic conductance (soil-to-leaf)	mol m^{-2} s^{-1} MPa^{-1}
Q	incident PAR	mol m^{-2} s^{-1}
V_{cmax}	carboxylation capacity	mol m^{-2} s^{-1}
Γ_*	CO2 compensation point	mol mol^{-1}
ϕ	drought factor	-
Ψ_0	critical leaf water potential	MPa (negative)
Ψ_{soil}	soil water potential	MPa (negative)

5. Symbols

Symbol	Variable	Unit
b	parametre of the soil water retention curve	-
K_{soil}	leaf area specific soil hydraulic conductance	mol m^{-2} s^{-1} MPa^{-1}
K_{xylem}	leaf area specific xylem hydraulic conductance	mol m^{-2} s^{-1} MPa^{-1}
k_{sat}	saturated bulk soil hydraulic conductivity	mol m^{-1} s^{-1} MPa^{-1}
L_v	root length density	m m^{-3}
LAI	leaf area index (all-sided)	-
r_{root}	mean radius of water-absorbing roots	m
R_1	root length index	m m^{-2}
Ψ_e	soil water potential at saturation	MPa (negative)

6. References

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7. Extra

The Unified Stomatal Optimization (USO) function for stomatal control as developed by Medlyn et al. (2011) looks like this:

$$g_s \approx g_0 + \left(1 + \frac{g_1}{\sqrt{D}}\right)\frac{A}{C_a}$$

Our model gives an interpretation to their parametre g_1 (at sufficiently high C_a and low g_0) as such:

$$g_1 = \sqrt{\frac{K|\Psi_0|}{1.6}\left(\frac{k_m}{V_{cmax}} + \frac{4\Gamma_*}{J(Q)}\right)}$$