Solving the energy balance equation in enerbil.f90

Jan Polcher

First written at the end of the 90's but not distributed at the time

In order to simplify the implementation of the implicit coupling between surface schemes and atmospheric models we propose here a new set of variables to be exchanged between the two models.

In Polcher et al. (1998) the proposed list of variables to be exchanged was made complicated by the fact that it had to leave a choice open between computing the surface drag in the surface scheme or the atmospheric model. Since publication of this paper a new formulation of the exchange variables was proposed which simplifies the variables to be exchanged and gives them clear physical meaning (Best et al., 2004).

In the present note the new variables to be exchanged are presented. We also discuss their application for solving the surface energy balance.

1 Simplified interface variables

In order to achieve this simplification of the exchanged data we start from equation 15 in Polcher et al. (1998) for the first level of the atmosphere. The aim is to obtain a linear relation between the surface fluxes and the temperature and specific humidity at the first level. The calculations are started with equation 15 re-written as follows :

$$\frac{X_1^{t+1} - X_1^t}{\Delta t} = \frac{1}{\Delta z_1}\left(K_{1+1/2}\frac{X_2^{t+1} - X_1^{t+1}}{\delta z_1} - F_{X,1/2}^{t+1}\right) \tag{1}$$

where X is potential enthalpy (θ) (As defined in equation 21 of Polcher et al. 1998.) or specific humidity (q) and K is the "eddy-diffusivity". To simplify this equation we need to replace X_2^{t+1} by its relation to the temperature on the lower level given in equation 16 of (Polcher et al., 1998). This information is available when the planetary boundary layer diffusion is solved with the method proposed by Richtmeyer and Morton (1967) . Some calculus yield the following relation between lowest atmospheric variables and the surface fluxes :

$$X_1^{t+1} = \mathcal{A}_X F_{X,1/2}^{t+1} + \mathcal{B}_X \tag{2}$$

using :

$$\gamma = 1 + \frac{\Delta t}{\Delta z_1}\frac{K_{1+1/2}}{\delta z_1}(1 - A_{X,2}^t) \tag{3}$$

$$\mathcal{A}_X = \frac{\Delta t}{\Delta z_1}\gamma^{-1} \tag{4}$$

$$\mathcal{B}_X = \left(\frac{\Delta t}{\Delta z_1}\frac{K_{1+1/2}}{\delta z_1}B_{X,2}^t + X_1^t\right)\gamma^{-1} \tag{5}$$

$\mathcal{A}_X$ and $\mathcal{B}_X$ do not contain any information on the surface condition at time $t+1$ and can thus be computed in the PBL prior to the land-surface processes. Passing these variables to the

land-surface schemes allows to solve elegantly the problem of the surface drag as $K_{1/2}$ is not used in their calculation. Thus these variables leave the choice open to compute the surface drag in the surface scheme or in the atmosphere.

Using equation 2 between first level atmospheric variables and surface fluxes allows to simplify the calculation of the surface fluxes in the case of an implicit coupling. For the other numerical methods nothing changes as the surface and the atmosphere are not solved together. Thus, $\mathcal{A}_X$ and $\mathcal{B}_X$ are not used.

Starting from the surface flux equation (20 in Polcher et al (1998)) we can derive a simplified equations for the implicit coupling :

$$F_{X,1/2}^{t+1} = \frac{K_{X,1/2}}{\delta z_0} \left(X_1^{t+1} - X_0^{t+1} \right) \tag{6}$$

replacing X_1^{t+1} by equation 2 we obtain :

$$
\begin{aligned}
F_{X,1/2}^{t+1} &= \frac{\mathcal{B}_X - X_0^{t+1}}{\frac{\delta z_0}{K_{X,1/2}} - \mathcal{A}_X} \tag{7} \\
&= \frac{\mathcal{B}_X - X_0^t}{\frac{\delta z_0}{K_{X,1/2}} - \mathcal{A}_X} - \frac{X_0^{t+1} - X_0^t}{\frac{\delta z_0}{K_{X,1/2}} - \mathcal{A}_X} \tag{8} \\
&= \tilde{F}_{X,1/2} - F_{X,1/2}' \Delta X_0 \tag{9}
\end{aligned}
$$

In equation 9 the new surface flux has been split into two components : The first one is based on the old surface conditions ($\tilde{F}_{X,1/2}$) and the evolving atmospheric state while the second one describes the sensitivity of the flux ($F_{X,1/2}'$) to the variations of the surface variables.

When $\mathcal{A}_X$ and $\mathcal{B}_X$ are passed to the land-surface scheme one can still fall back to the explicit coupling. This is the case when the variations of the atmospheric conditions induced by the surface fluxes are not taken into account when solving the surface energy balance. It is achieved by choosing the following values for $\mathcal{A}_X$ and $\mathcal{B}_X$:

$$
\begin{aligned}
\mathcal{A}_X &= 0 \tag{10} \\
\mathcal{B}_X &= X_1^t \tag{11}
\end{aligned}
$$

As in this case the atmospheric variables are not allowed to adjust to the new surface fluxes the interpretation of the equation 9 is simpler. It is easily recognised that the first term is the flux obtained for the old surface and atmospheric conditions ($\tilde{F}_{X,1/2}$) and the second one a correction term ($F_{X,1/2}'$) linked to the variations of surface conditions when integrating from t to $t+1$.

2 Solving the surface energy balance equation for an implicit coupling

The unstressed surface energy balance is written as :

$$C_s \frac{\theta_0^{t+1} - \theta_0^t}{\Delta t} = \frac{C_s}{\Delta t} \Delta \theta_0 = R_n^{t+1} + H^{t+1} + f_s L_s E^{t+1} + (1 - f_s) L_e E^{t+1} + G \tag{12}$$

where E is the potential evaporation, f_s the snow fraction and L_s the latent heat of sublimation and L_e the latent heat of evaporation. All other variables have their standard meaning !

All the fluxes in the above equation which have a dependence on the surface temperature need to be simplified so that the energy balance can be solved without any iteration. Their dependence on atmospheric variables needs to be expanded as in equation 9 to make an implicit coupling possible.

2.1 Net surface radiation :

The aim is to achieve a linear approximation of the net surface radiation for the new surface temperature. Using a limited Taylor series of the emitted long wave radiation we obtain an estimation of net radiation for the new surface temperature :

$$
\begin{align}
R_n^{t+1} &= (1-a)S_\Downarrow + Lw_\Downarrow - \epsilon\sigma\left(\frac{\theta_0^{t+1}}{c_p}\right)^4 \tag{13}\\
&\simeq (1-a)S_\Downarrow + Lw_\Downarrow - \epsilon\sigma\left(\frac{\theta_0^t}{c_p}\right)^4 - \frac{4\epsilon\sigma}{c_p}\left(\frac{\theta_0^t}{c_p}\right)^3(\theta_0^{t+1} - \theta_0^t) \tag{14}\\
&\simeq \tilde{R}_n - \frac{4\epsilon\sigma}{c_p}\left(\frac{\theta_0^t}{c_p}\right)^3(\theta_0^{t+1} - \theta_0^t) \tag{15}\\
&\simeq \tilde{R}_n - R_n'\Delta\theta_0 \tag{16}
\end{align}
$$

2.2 Sensible heat flux :

This flux is simplified using equation 7 :

$$
\begin{align}
H^{t+1} &= \frac{K_{\theta,1/2}}{\delta z_0}(\theta_1^{t+1} - \theta_0^{t+1}) \tag{17}\\
&= \frac{\mathcal{B}_\theta - \theta_0^{t+1}}{\frac{\delta z_0}{K_{\theta,1/2}} - \mathcal{A}_\theta} \tag{18}\\
&= \frac{\mathcal{B}_\theta - \theta_0^t}{\frac{\delta z_0}{K_{\theta,1/2}} - \mathcal{A}_\theta} - \frac{(\theta_0^{t+1} - \theta_0^t)}{\frac{\delta z_0}{K_{\theta,1/2}} - \mathcal{A}_\theta} \tag{19}\\
&= \tilde{H} - H'\Delta\theta_0 \tag{20}
\end{align}
$$

2.3 Potential evaporation :

In order to solve for the potential evaporation of the next time step, we have to linearise with a limited Taylor expansion the saturated humidity at the surface temperature. This can be written as :

$$
\begin{align}
q_s(\theta_0^{t+1}) &\simeq q_s(\theta_0^t) + \left.\frac{\partial q_s}{\partial\theta}\right|_{\theta_0^t}\frac{1}{c_p}(\theta_0^{t+1} - \theta_0^t) \tag{21}\\
&\simeq q_s(\theta_0^t) + \frac{Q'}{c_p}\Delta\theta_0 \tag{22}
\end{align}
$$

From this follows the implicit formulation for potential evaporation.

$$
E^{t+1} = \frac{K_{q,1/2}}{\delta z_0}(q_1^{t+1} - q_s(\theta_0^{t+1})) \tag{23}
$$

$$= \frac{\mathcal{B}_q - q_s(\theta_0^{t+1})}{\frac{\delta z_0}{K_{q,1/2}} - \mathcal{A}_q} \tag{24}$$

$$= \frac{\mathcal{B}_q - q_s(\theta_0^t) - \frac{Q'}{c_p}\Delta\theta_0}{\frac{\delta z_0}{K_{q,1/2}} - \mathcal{A}_q} \tag{25}$$

$$= \frac{\mathcal{B}_q - q_s(\theta_0^t)}{\frac{\delta z_0}{K_{q,1/2}} - \mathcal{A}_q} - \frac{\frac{Q'}{c_p}\Delta\theta_0}{\frac{\delta z_0}{K_{q,1/2}} - \mathcal{A}_q} \tag{26}$$

$$= \tilde{E} - E'\Delta\theta_0 \tag{27}$$

It has to be noted that if E is reduced by an α factor (i.e. $E^{t+1} = \frac{K_{q,1/2}}{\delta z_0}(q_1^{t+1} - \alpha q_s(\theta_0^{t+1})))$) the same reasoning as above holds and the following terms are obtained for E :

$$\tilde{E} = \frac{\mathcal{B}_q - \alpha q_s(\theta_0^t)}{\frac{\delta z_0}{K_{q,1/2}} - \mathcal{A}_q} \tag{28}$$

$$E' = \frac{\frac{\alpha Q'}{c_p}\Delta\theta_0}{\frac{\delta z_0}{K_{q,1/2}} - \mathcal{A}_q} \tag{29}$$

Using Equation 27 the sum of latent heat fluxes ($LE = f_s L_s E^{t+1} + (1-f_s)L_e E^{t+1}$) used in the unstressed surface energy balance (12) will be written as :

$$LE = f_s L_s(\tilde{E} - E'\Delta\theta_0) + (1-f_s)L_e(\tilde{E} - E'\Delta\theta_0) \tag{30}$$

At this stage all fluxes which include a surface temperature dependency and a coupling to the atmosphere have been split into two components. The first term only depends on the old surface temperature or surface saturation humidity while the other expresses the sensitivity of the flux to the surface temperature variation. The unstressed surface energy balance (Equation 12) can now be rewritten in a form which can easily be solved to obtain the surface temperature tendency :

$$\frac{C_s}{\Delta t}\Delta\theta_0 = \tilde{R}_n + \tilde{H} + f_s L_s \tilde{E}_s + (1-f_s)L_e\tilde{E} + G - (R_n' + H' + f_s L_s E_s' + (1-f_s)L_e E')\Delta\theta_0 \tag{31}$$

2.4 Introducing the stress functions

The above derivation makes the assumption that the potential evaporation is computed using a bulk formulation. This is a rather strong assumption which we should be able to relax in ORCHIDEE at some point. It has been shown for instance that the bulk formulation overestimates potential evaporation and is more sensitive to climate change than the Penman-Monteith formulation (Barella-Ortiz et al., 2013). It is thus of interest to be able to replace in the standard surface energy balance equation the potential evaporation by the one obtained from an unstressed surface energy balance equation (Equation 12) (see Barella-Ortiz et al. 2013 for further details). In order to achieve this we will operate by analogy when introducing the stress functions. In all generality we have one for sublimation (β_s) and one for evaporation (β). This corresponds to taking into account the stress functions in a time explicit way.

The standard surface energy balance equation is written :

4

$$C_s \frac{\theta_0^{t+1} - \theta_0^t}{\Delta t} = R_n^{t+1} + H^{t+1} + f_s L_s \beta_s E^{t+1} + (1 - f_s) L_e \beta E^{t+1} + G \tag{32}$$

Applying these stress function to the implicit formulation of the unstressed surface energy balance equation we obtain :

$$\frac{C_s}{\Delta t} \Delta\theta_0 = \tilde{R}_n + \tilde{H} + f_s L_s \beta_s \tilde{E}_s + (1 - f_s) L_e \beta \tilde{E} + G - (R'_n + H' + f_s L_s \beta_s E'_s + (1 - f_s) L_e \beta E') \Delta\theta_0 \tag{33}$$

3 The previous method for introducing the stress functions

In SECHIBA the beta function was directly integrated in the implicit resolution of the latent heat flux. In order to illustrate the difference the equations used previously are derived with the nomenclature used in this document.

Evaporation (E_v) is formulated directly with the stress function :

$$E_v^{t+1} = \frac{K_{q,1/2}}{\delta z_0} \beta(q_1^{t+1} - \alpha q_s(\theta_0^{t+1})) \tag{34}$$

Using directly equation 2 for moisture we obtain the following implicit formulation for evaporation :

$$E^{t+1} = \frac{\mathcal{B}_q - \alpha q_s(\theta_0^{t+1})}{\frac{\delta z_0}{\beta K_{q,1/2}} - \mathcal{A}_q} \tag{35}$$

$$= \frac{\mathcal{B}_q - \alpha q_s(\theta_0^t) - \frac{\alpha Q'}{c_p} \Delta\theta_0}{\frac{\delta z_0}{\beta K_{q,1/2}} - \mathcal{A}_q} \tag{36}$$

$$= \frac{\mathcal{B}_q - \alpha q_s(\theta_0^t)}{\frac{\delta z_0}{\beta K_{q,1/2}} - \mathcal{A}_q} - \frac{\frac{\alpha Q'}{c_p} \Delta\theta_0}{\frac{\delta z_0}{\beta K_{q,1/2}} - \mathcal{A}_q} \tag{37}$$

$$= \tilde{E}_v - E'_v \Delta\theta_0 \tag{38}$$

This last equation can now be used to replace the potential evaporation in equation 12 by using the appropriate β functions for evaporation and sublimation.

Because in this formulation potential evaporation is not clearly identified anymore, it was chosen at the time ORCHIDEE was developed, to keep β as a multiplying factor for $\tilde{E}$ and E'.

4 Forcing the surface with a relaxation to observations

Implemented in dim2_driver.f90 but not tested

Forcing a land-surface scheme directly with observed atmospheric variables may lead to numerical problems as the consistency between simulated fluxes and atmospheric conditions is lost. One remedy to this problem is to simulate atmospheric conditions as a result of surface fluxes and a Newtonian relaxation towards observations.

The evolution equation for atmospheric conditions can easily be obtained with :

$$\frac{X_1^{t+1} - X_1^t}{\Delta t} = \frac{1}{\Delta z_r} \left(K_r \frac{X_{obs}^{t+1} - X_1^{t+1}}{\delta z_r} - F_{X,1/2}^{t+1} \right) \tag{39}$$

with δz_r is the thickness of the relaxation layer. It follows that $\Delta z_r = \frac{1}{2}(\delta z_0 + \delta z_r)$. X_{obs}^{t+1} is the observed variable towards we will relax at time step $t+1$. In order to simplify the discussion we will set $\frac{K_r}{\delta z_r} = \Lambda$ and $\Delta z_r = \frac{1}{2}\delta z_0$.

Solving equation 39 with the same approach as in equation 2 we obtain :

$$X_1^{t+1} \quad = \quad \mathcal{A}_X F_{X,1/2}^{t+1} + \mathcal{B}_X \tag{40}$$

using :

$$\mathcal{A}_X \quad = \quad \frac{1}{\frac{\delta z_0}{2\Delta t} + \Lambda} \tag{41}$$

$$\mathcal{B}_X \quad = \quad \frac{\Lambda X_{obs}^{t+1}}{\frac{\delta z_0}{2\Delta t} + \Lambda} + \frac{X_1^t}{1 + \frac{2\Delta t}{\delta z_0}\Lambda} \tag{42}$$

Using the above equation the two extreme cases of the relaxation can be discussed.

Strongly constrained forcing : To achieve a forcing of the land-surface scheme which follows closely the observed atmospheric variables large values of the relaxation constant needs (Λ) to be chose. In this case the following relations are obtained :

$$\Lambda \to \infty \Rightarrow \left\{ \begin{array}{l} \mathcal{A}_X \to 0 \\ \mathcal{B}_X \to X_{obs}^{t+1} \end{array} \right. \Rightarrow X_1^{t+1} = X_{obs}^{t+1} \tag{43}$$

Excluding observations : The other extreme of the relaxation is to suppress the influence of observed atmospheric condition on the forcing variables. To achieve this case the relaxation constant needs to be very small :

$$\Lambda \to 0 \Rightarrow \left\{ \begin{array}{l} \mathcal{A}_X \to 1 \\ \mathcal{B}_X \to X_1^t \end{array} \right. \Rightarrow X_1^{t+1} = X_1^t \tag{44}$$

Does not work properly. The time filtering induces a time shift and dampens strongly the diurnal cycle! To avoid the last effect vales of more than 1000 are needed for the relaxation constant. Probably a higher order scheme is needed.

References

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