Description of the interface with the surface and the computation of the turbulent diffusion in LMDZ

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4.4. Calculation of surface temperature and surface heat fluxes in ORCHIDEE coupled with LMDZ (section 1.5.2 in *Dufresne and Ghattas* [2009])

4.4.1. Surface energy balance:

The soil surface temperature 'Ts' is controlled by the surface energy balance equation [E.Q. A29, in Hourdin F. (1992)]:

$$\mathrm{Cs}\frac{T_S^t - T_S^{t-\delta t}}{\delta t} = \mathrm{Fs} + \sum F^{\downarrow}(T_S^t)\ (\mathrm{A29})$$

where Cs is the heat capacity of the surface, Fs represents the heat transfer from the surface to the deepest layers, $\sum F^{\downarrow}(T_S^t)$ represents the balance of net fluxes on land surface, e.g., the sum of net radiation at the ground, the sensible heat flux and latent heat flux. $\sum F^{\downarrow}(T_S^t)$ is composed of three terms [e.q. (61), in Dufresne and Ghattas, (2009)]:

$$\sum F^{\downarrow}(T_S^t) = F = F_{rad} + F_1^h + LF_1^q (61)$$

$$\xRightarrow{(\mathbf{A29}),(\mathbf{61})} \mathrm{Cs}\frac{T_S^t - T_S^{t-\delta t}}{\delta t} = \mathrm{Fs} + F_{rad} + F_1^h + LF_1^q$$

The above flux terms are introduced in the following sections 4.4.2 (F_1^h), 4.4.3 (LF_1^q), and 4.4.4 (F_{rad}).

4.4.2. Enthalpy:

(1) The basic equations:

$$F_1^H = K_1^h\left(H_1 - C_p\left(\frac{P_r}{P}\right)^{\kappa} T_s\right) (62)$$

The coefficient K_1^h is written in a "bulk" type formula:

$$K_1^q = K_1^h = K_1 = \rho|\overline{V}|C_d\left(\text{unit:} \frac{kg}{m^3} \times \frac{m}{s} = \frac{kg}{m^2 \times s} = \frac{mm}{s}\right)$$

By replacing H_1 with Equation (54) in Dufresne and Ghattas, (2009):

$$H_1 = A_1^h + B_1^h \times F_1^H \times \delta t$$

we obtain [e.q. (64)-(66) in Dufresne and Ghattas, (2009)]:

$$F_1^H = M_1^h + N_1^h T_s (64)$$

with

$$M_1^h = \frac{K_1^h A_1^h}{1 - K_1^h B_1^h \delta t} (65)$$

$$N_1^h = \frac{-K_1^h C_p \left(\frac{P_r}{P}\right)^{\kappa}}{1 - K_1^h B_1^h \delta t} (66)$$

F_1^H can be divided into two terms: the value at old time step ($\{T_s\}$) and a sensitivity term ($T_s - \{T_s\}$) (Truncated Taylor expansions). And, in both of orchidee and boundary layer, the surface pressure and reference level pressure equal the first level pressure of LMDZ. Then, E.q. (64) can also be written as:

$$F_1^H = M_1^h + N_1^h T_s = \frac{K_1^h A_1^h}{1 - K_1^h B_1^h \delta t} + \frac{-K_1^h C_p \left(\frac{P_r}{P}\right)^{\kappa}}{1 - K_1^h B_1^h \delta t} T_s = \frac{K_1^h A_1^h - K_1^h C_p \left(\frac{P_r}{P}\right)^{\kappa} T_s}{1 - K_1^h B_1^h \delta t}$$

$$\xrightarrow{\left(\frac{P_r}{P}\right)^{\kappa}\approx 1} F_1^H = \frac{K_1^h A_1^h - K_1^h C_p T_s}{1 - K_1^h B_1^h \delta t} = \frac{A_1^h - C_p T_s}{\frac{1}{K_1^h} - B_1^h \delta t} = \frac{A_1^h - C_p\{T_s\}}{\frac{1}{K_1^h} - B_1^h \delta t} - \frac{C_p}{\frac{1}{K_1^h} - B_1^h \delta t}(T_s - \{T_s\})$$

(2) The equations in the source code and its correlation with the equations here:

$$F_1^H = \text{sensfl}_{\text{old}} + (-C_p) \times \text{sensfl\_sns} \times (T_s - \{T_s\})$$

The $\text{sensfl}_{\text{old}}$ and sensfl_sns are the 'old' term and the 'sensitivity' term, respectively.

The 'old' term:

$$\text{sensfl}_{\text{old}} = \frac{\text{petBcoef} - \text{psold}}{\text{zikt} - \text{petAcoef}} = M_1^h + N_1^h\{T_s\} \left(\text{unit: } \frac{\frac{J}{kg}}{\frac{m^3}{kg}\frac{s}{m}} = \frac{J}{kg} \times \frac{kg}{m^2 \times s} = \frac{W}{m^2}\right)$$

$$\text{psold} = C_p\{T_s\} = \text{cp\_air} \times \text{temp\_air} \left(\text{unit:} \frac{J}{kg \times K} \times K = \frac{J}{kg}\right)$$

$$\text{zikt} = \frac{1}{K_1^h}, \text{petBcoef} = A_1^h, \text{petAcoef} = B_1^h \delta t, \text{temp}_{\text{air}} = \{T_s\}, \text{cp}_{\text{air}} = C_p$$

The 'sensitivity' term:

$$\text{sensfl}_{\text{sns}} = \frac{1}{\text{zikt} - \text{petAcoef}} = \frac{N_1^h}{-C_p} \left(\text{unit: } \frac{kg}{m^3} \times \frac{m}{s} = \frac{kg}{m^2 \times s}\right)$$

$$N_1^h = \frac{-K_1^h C_p \left(\frac{P_r}{P}\right)^{\kappa}}{1 - K_1^h B_1^h \delta t} = \frac{-C_p \left(\frac{P_r}{P}\right)^{\kappa}}{\frac{1}{K_1^h} - B_1^h \delta t} \xrightarrow{\left(\frac{P_r}{P}\right)^{\kappa}\approx 1} N_1^h = -C_p \times \frac{1}{\frac{1}{K_1^h} - B_1^h \delta t}$$

4.4.3. Water vapor flux:

(1) Basic equations:

$$F_1^q = \beta K_1^q (Q - q_0) (67)$$

With β the coefficient of evaporation which accounts the relationship between actual evaporation and potential evaporation, and q_0 is the saturated specific humidity corresponding to T_s, they are bounded by the Clausius-Clapeyron equation:

$$q_0 = q_{sat}(T_s) (68)$$

The dependence of the saturated specific humidity as a function of temperature is linearized with respect to the value at beginning of the time step:

$$\delta q = q_0 - q_{sat}(\{T_s\}) = \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} \times \delta T_s = \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} \times \frac{1}{C_P} \times \delta H \Rightarrow q_0 = q_{sat}(\{T_s\}) + \frac{1}{C_P} \times \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} \times \delta H$$

e.q. (69) in Dufresne and Ghattas (2009):

$$\Rightarrow q_0 = q_{sat}(\{T_s\}) + \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} (T_s - \{T_s\}) (69)$$

By replacing Q in equation (67) by its expression Eq. (54) in Dufresne and Ghattas, (2009)

$$Q = A_1^h + B_1^h \times F_1^q \times \delta t$$

which takes into account the variation of the specific humidity and the linearization of saturated specific humidity with surface temperature (Eq. 69), we then obtain [e.q.(70)-(72) in Dufresne and Ghattas, (2009)]::

$$F_1^Q = M_1^q + N_1^q T_s (70)$$

With

$$M_1^q = \frac{\beta K_1^q \left(A_1^q - \{q_{sat}\} + \{T_s\}\frac{\partial q_{sat}}{\partial T}\right)}{1 - \beta K_1^q B_1^q \partial t} (71)$$

$$N_1^q = \frac{-\beta K_1^q \frac{\partial q_{sat}}{\partial T}}{1 - \beta K_1^q B_1^q \partial t} (72)$$

F_1^q is divided into two terms: the value at old time step ($\{T_s\}$) and the sensitivity term $(T_s - \{T_s\})$. Then, E.q. (70) can be also written as:

$$F_1^Q = M_1^q + N_1^q T_s = \frac{\beta K_1^q \left(A_1^q - \{q_{sat}\} + \{T_s\}\frac{\partial q_{sat}}{\partial T}\right)}{1 - \beta K_1^q B_1^q \delta t} + \frac{-\beta K_1^q \frac{\partial q_{sat}}{\partial T}}{1 - \beta K_1^q B_1^q \partial t} T_s$$

$$= \frac{\beta K_1^q \left(A_1^q - \{q_{sat}\} + \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} \{T_s\} - \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} T_s\right)}{1 - \beta K_1^q B_1^q \delta t}$$

$$\xrightarrow{\{T_s\} \neq T_s} F_1^Q = \frac{\beta K_1^q (A_1^q - \{q_{sat}\})}{1 - \beta K_1^q B_1^q \delta t} - \frac{\beta K_1^q \left[\left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} (T_s - \{T_s\})\right]}{1 - \beta K_1^q B_1^q \delta t} = \frac{\beta (A_1^q - \{q_{sat}\})}{\frac{1}{K_1^q} - \beta B_1^q \delta t} - \frac{\beta \left[\left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} (T_s - \{T_s\})\right]}{\frac{1}{K_1^q} - \beta B_1^q \delta t}$$

$$= \frac{\beta \left(A_1^q - \left[\{q_{sat}\} + \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} (T_s - \{T_s\})\right]\right)}{\frac{1}{K_1^q} - \beta B_1^q \delta t} = \frac{\beta (A_1^q - \text{q\_sat\_new})}{\frac{1}{K_1^q} - \beta B_1^q \delta t}$$

The total evaporation is divided into sublimation and evaporation:

Sublimation:

$$F_{sub,1}^Q = M_{sub,1}^q + N_{sub,1}^q T_s = \frac{\beta_{sub} \left(A_1^q - \left[\{q_{sat}\} + \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} (T_s - \{T_s\})\right]\right)}{\frac{1}{K_1^q} - \beta_{sub} B_1^q \delta t}$$

Evaporation:

$$F_{eva,1}^Q = M_{eva,1}^q + N_{eva,1}^q T_s = \frac{\beta_{eva} \left(A_1^q - \left[\{q_{sat}\} + \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}} (T_s - \{T_s\})\right]\right)}{\frac{1}{K_1^q} - \beta_{eva} B_1^q \delta t}$$

(2) The source code (SUBROUTINE enerbil_surftemp) and its correlation with the equations here:

$$F_1^Q = \left(\frac{1}{L}\right) \times (\text{larsub}_{old} + \text{lareva}_{old}) + \left(\frac{-C_p}{L}\right) \times (\text{larsub}_{sns} + \text{lareva}_{sns}) \times (T_s - \{T_s\})$$

$$L = \begin{cases} \text{chalsu0, ice} \\ \text{chalev0, water} \end{cases}$$

The saturated specific humidity:

$$\text{qsol}_{\text{sat}_{new}} = \text{qsol}_{sat} + \text{zicp} \times \text{pdqsold} \times \text{dtheta} = \text{qsol}_{sat} + \text{zicp} \times \text{dev}_{qsol} \times \frac{\mathbf{pb^{kappa}}}{\mathbf{cp\_air}} \times \text{dtheta} \left(\text{unit: } \frac{\mathbf{kg}}{\mathbf{kg}}\right)$$

$$\text{pdqsold} = \text{dev\_qsol} \times \frac{\mathbf{pb^{kappa}}}{\mathbf{cp\_air}} \left(\text{unit: } \frac{\mathbf{1}}{\mathbf{K}}?\right)$$

Problems in the source code here. 'pdqsold' changes to $\rightarrow$ pdqsold = dev_qsol

The 'old' term:

$$\text{larsub}_{old} = \text{chalsu0} \times \text{vbeta1} \times (1 - \text{vbeta5}) \times \frac{\text{peqBcoef} - \text{qsol}_{sat}}{\text{zikq} - \text{peqAcoef}} = L(M_{sub,1}^q + N_{sub,1}^q \{T_s\}) \left(\text{unit: } \frac{W}{m^2}\right)$$

$$\mathrm{zikq} = \frac{1}{K_1^q}, \mathrm{peqBcoef} = A_1^q, \mathrm{petAcoef} = B_1^q\,\partial t$$

$$\mathrm{lareva_{old}} = \mathrm{chalev0} \times (1-\mathrm{vbeta1}) \times (1-\mathrm{vbeta5}) \times \mathrm{vbeta} \times \frac{\mathrm{peqBcoef} - \mathrm{valpha} \times \mathrm{qsol_{sat}}}{\mathrm{zikq} - \mathrm{peqAcoef}} + \mathrm{chalev0} \times \mathrm{vbeta5} \times \frac{\mathrm{peqBcoef} - \mathrm{qsol_{sat}}}{\mathrm{zikq} - \mathrm{peqAcoef}} = L\left(M_{eva,1}^q + N_{eva,1}^q\{T_s\}\right) \left(\text{unit: } \frac{J}{kg} \times \frac{\frac{kg}{kg}}{\frac{m^3}{kg}\frac{s}{m}} = \frac{W}{m^2}\right)$$

The 'sensitivity' term:

e.q. (72) in Dufresne and Ghattas, (2009):

$$N_{sub,1}^q = \frac{-\beta_{sub} K_1^q \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}}}{1 - \beta_{sub} K_1^q B_1^q \delta t} = \frac{-\beta_{sub} \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}}}{\frac{1}{K_1^q} - \beta_{sub} B_1^q \delta t}$$

$$N_{eva,1}^q = \frac{-\beta_{eva} K_1^q \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}}}{1 - \beta_{eva} K_1^q B_1^q \delta t} = \frac{-\beta_{eva} \left.\frac{\partial q_{sat}}{\partial T}\right|_{\{T_s\}}}{\frac{1}{K_1^q} - \beta_{eva} B_1^q \delta t}$$

In code, SUBROUTINE enerbil_surftemp:

$$\mathrm{larsub_{sns}} = \mathrm{chalsu0} \times \mathrm{vbeta1} \times (1-\mathrm{vbeta5}) \times (-\mathrm{zicp}) \times \frac{-\mathrm{pdqsold}}{\mathrm{zikq} - \mathrm{peqAcoef}} = \frac{L N_1^q}{-C_p} \left(\text{unit: } \frac{kg}{m^2 \times s}\right)$$

$$\mathrm{zicp} = \frac{1}{\mathrm{cp\_air}} \left(\text{unit: } \frac{kg \times K}{J}\right)$$

$$\mathrm{lareva_{sns}} = \mathrm{chalev0} \times [(1-\mathrm{vbeta1}) \times (1-\mathrm{vbeta5}) \times \mathrm{vbeta} \times \mathrm{valph} + \mathrm{vbeta5}] \times (-\mathrm{zicp}) \times \frac{-\mathrm{pdqsold}}{\mathrm{zikq} - \mathrm{peqAcoef}} = \frac{L N_1^q}{-C_p} \left(\text{unit: } \frac{J}{kg} \times \frac{\frac{kg \times K}{J} \times \frac{1}{K}}{\frac{m^3}{kg} \times \frac{s}{m}} = \frac{kg}{m^2 \times s}\right)$$

Problems in the source code: β missed in denominator.

4.4.4. Net radiation

$$F_{rad} = Rsw_{net} + Rlw_{down} - [\varepsilon\sigma(T_S^t)^4 + (1-\varepsilon) \times Rlw_{down}]$$

F_1^q is divided into two terms: the value at old time step and the sensitivity term. Then, it can also be written as:

$$F_{rad} = Rsw_{net} + Rlw_{down} - \left[\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + (1-\varepsilon) \times Rlw_{down}\right] + \varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 - \varepsilon\sigma(T_S^t)^4$$

$$\approx Rsw_{net} + Rlw_{down} - \left[\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + (1-\varepsilon) \times Rlw_{down}\right] + \varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 - \varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 - 4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3\left(T_S^t - T_S^{t-\delta t}\right)$$

$$= \left\{Rsw_{net} + Rlw_{down} - \left[\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + (1-\varepsilon) \times Rlw_{down}\right]\right\} - \left\{4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3\left(T_S^t - T_S^{t-\delta t}\right)\right\}$$

The 'old' term, in Subroutine enerbil_begin:

$$\mathrm{netrad} = \mathrm{lwdown} + \mathrm{swnet} - [\mathrm{emis} \times c_{stefan} \times \mathrm{temp\_sol}^4 + (1-\mathrm{emis}) \times \mathrm{lwdown}] \left(\text{unit:} \frac{W}{m^2}\right)$$

The 'sensitivity' term, in Subroutine enerbil_flux:

$$\mathrm{lwup} = [\mathrm{emis} \times c_{stefan} \times \mathrm{temp\_sol}^4 + (1-\mathrm{emis}) \times \mathrm{lwdown}] + [\mathrm{quatre} \times \mathrm{emis} \times c_{stefan} \times \mathrm{temp\_sol}^3 \times (\mathrm{temp\_sol\_new} - \mathrm{temp\_sol})] \left(\text{unit: } \frac{W}{m^2}\right)$$

$$\mathrm{netrad} = \mathrm{lwdown} + \mathrm{swnet} - \mathrm{lwup}$$

$$\mathrm{netrad\_sns} = \mathrm{zicp} \times \mathrm{quatre} \times \mathrm{emis} \times c_{\mathrm{stefan}} \times [(\mathrm{zicp} \times \mathrm{psold})^3]$$

$$= \mathrm{zicp} \times \mathrm{quatre} \times \mathrm{emis} \times c_{\mathrm{stefan}} \times \mathrm{temp\_sol}^3 \left(\mathrm{unit:}\ \frac{\mathrm{kg} \times \mathrm{K}}{\mathrm{J}} \times \frac{\mathrm{W}}{\mathrm{m}^2 \times \mathrm{K}^4} \times \mathrm{K}^3 = \frac{\mathrm{kg}}{\mathrm{m}^2 \times \mathrm{s}}\right)$$

4.4.5. The sum of old term and the sensitivity term

(1) The old term

$$\mathrm{sum_{old}} = \mathrm{Fs} + \mathrm{Rsw_{net}} + \mathrm{Rlw_{down}} - \left[\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + (1-\varepsilon) \times \mathrm{Rlw_{down}}\right] + M_1^h + N_1^h T_S^{t-\delta t} + LM_1^q + LN_1^q T_S^{t-\delta t}$$

In code: SUBROUTINE enerbil_surftemp:

$$\mathrm{sum_{old}} = \mathrm{netrad} + \mathrm{sensfl_{old}} + \mathrm{larsub_{old}} + \mathrm{lareva_{old}} + \mathrm{soilflx}$$

$$= \mathrm{netrad} + \frac{\mathrm{petBcoef} - \mathrm{psold}}{\mathrm{zikt} - \mathrm{petAcoef}} + \mathrm{chalsu0} \times \mathrm{vbeta1} \times (1 - \mathrm{vbeta5}) \times \frac{\mathrm{peqBcoef} - \mathrm{qsol_{sat}}}{\mathrm{zikq} - \mathrm{peqAcoef}}$$

$$+ \mathrm{chalev0} \times (1 - \mathrm{vbeta1}) \times (1 - \mathrm{vbeta5}) \times \mathrm{vbeta} \times \frac{\mathrm{peqBcoef} - \mathrm{valpha} \times \mathrm{qsol_{sat}}}{\mathrm{zikq} - \mathrm{peqAcoef}}$$

$$+ \mathrm{chalev0} \times \mathrm{vbeta5} \times \frac{\mathrm{peqBcoef} - \mathrm{qsol_{sat}}}{\mathrm{zikq} - \mathrm{peqAcoef}} + \mathrm{soilflx}$$

$$= \mathrm{netrad} + \frac{\mathrm{petBcoef} - \mathrm{psold}}{\mathrm{zikt} - \mathrm{petAcoef}} + \frac{\mathrm{peqBcoef} - \mathrm{qsol_{sat}}}{\mathrm{zikq} - \mathrm{peqAcoef}}$$

$$\times [\mathrm{chalsu0} \times \mathrm{vbeta1} \times (1 - \mathrm{vbeta5}) + \mathrm{chalev0} \times (1 - \mathrm{vbeta1}) \times (1 - \mathrm{vbeta5}) \times \mathrm{vbeta}$$

$$+ \mathrm{chalev0} \times \mathrm{vbeta5}] + \mathrm{soilflx} \left(\mathrm{unit:} \frac{\mathrm{W}}{\mathrm{m}^2}\right)$$

(2) The sensitivity term

$$\mathrm{sum_{sns}} = \frac{-4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3 + N_1^h + LN_1^q}{-\mathrm{cp_{air}}}$$

$$-\mathrm{cp_{air}} \times \mathrm{sum_{sns}} = N_1^h + LN_1^q - 4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3$$

In code: SUBROUTINE enerbil_surftemp:

$$\mathrm{sum_{sns}} = \mathrm{netrad_{sns}} + \mathrm{sensfl_{sns}} + \mathrm{larsub_{sns}} + \mathrm{lareva_{sns}}$$

$$= \mathrm{netrad_{sns}} + \frac{1}{\mathrm{zikt} - \mathrm{petAcoef}} + \mathrm{chalsu0} \times \mathrm{vbeta1} \times (1 - \mathrm{vbeta5}) \times (-\mathrm{zicp})$$

$$\times \frac{-\mathrm{pdqsold}}{\mathrm{zikq} - \mathrm{peqAcoef}} + \mathrm{chalev0} \times [(1 - \mathrm{vbeta1}) \times (1 - \mathrm{vbeta5}) \times \mathrm{vbeta} \times \mathrm{valph} + \mathrm{vbeta5}]$$

$$\times (-\mathrm{zicp}) \times \frac{-\mathrm{pdqsold}}{\mathrm{zikq} - \mathrm{peqAcoef}}$$

$$= -\mathrm{zicp} \times [-(\mathrm{quatre} \times \mathrm{emis} \times c_{\mathrm{stefan}} \times \mathrm{temp\_sol}^3)] + \frac{1}{\mathrm{zikt} - \mathrm{petAcoef}} + (-\mathrm{zicp})$$

$$\times \frac{-\mathrm{pdqsold}}{\mathrm{zikq} - \mathrm{peqAcoef}}$$

$$\times \{\mathrm{chalsu0} \times \mathrm{vbeta1} \times (1 - \mathrm{vbeta5}) + \mathrm{chalev0}$$

$$\times [(1 - \mathrm{vbeta1}) \times (1 - \mathrm{vbeta5}) \times \mathrm{vbeta} \times \mathrm{valph} + \mathrm{vbeta5}]\} \left(\mathrm{unit:} \frac{\mathrm{kg}}{\mathrm{m}^2 \times \mathrm{s}}\right)$$

4.4.6. Temperature and surface fluxes:

(1) The surface energy balance equation [e.q. (61) in Dufresne and Ghattas, (2009)]:

$$\sum F^{\downarrow}(T_S^t) = F = F_{\mathrm{rad}} + F_1^h + LF_1^q (61)$$

$$= \mathrm{Rsw_{net}} + \mathrm{Rlw_{down}} - [\varepsilon\sigma(T_S^t)^4 + (1-\varepsilon) \times \mathrm{Rlw_{down}}] + \left[M_1^h + N_1^h T_S^t\right] + L\left[M_1^q + N_1^q T_S^t\right]$$

$$\overset{(A29),(61)}{\Longrightarrow} Cs\frac{T_S^t - T_S^{t-\delta t}}{\delta t}$$

$$\approx Fs + Rsw_{net} + Rlw_{down} - \left[\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + 4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3\left(T_S^t - T_S^{t-\delta t}\right) + (1-\varepsilon)\times Rlw_{down}\right]$$

$$+ \left[M_1^h + N_1^h T_S^t\right] + L\left[M_1^q + N_1^q T_S^t\right]$$

(2) The variation of enthalpy with surface temperature: (variable 'dtheta' in the code)

$$\Rightarrow Cs\frac{T_S^t - T_S^{t-\delta t}}{\delta t} + 4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3\left(T_S^t - T_S^{t-\delta t}\right) - N_1^h\left(T_S^t - T_S^{t-\delta t}\right) - LN_1^q\left(T_S^t - T_S^{t-\delta t}\right)$$

$$\approx Fs + Rsw_{net} + Rlw_{down} - \left[\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + (1-\varepsilon)\times Rlw_{down}\right] + \left[M_1^h + N_1^h T_S^{t-\delta t}\right]$$

$$+ L\left[M_1^q + N_1^q T_S^{t-\delta t}\right] \Rightarrow \left(T_s^t - T_s^{t-\delta t}\right)\left(\frac{Cs}{\delta t} + cp_{air}\times sum_{sns}\right) = sum_{old} \Rightarrow T_s^t - T_s^{t-\delta t}$$

$$= \frac{sum_{old}}{\frac{Cs}{\delta t} + cp_{air}\times sum_{sns}}$$

$$\Rightarrow dtheta = C_p\left(T_s^t - T_s^{t-\delta t}\right) = \frac{C_p \times sum_{old}}{\frac{Cs}{\delta t} + cp_{air}\times sum_{sns}} = \frac{\delta t\times sum_{old}}{\frac{Cs}{C_p} + \delta t\times sum_{sns}}$$

In code, SUBROUTINE enerbil_surftemp:

$$dtheta = \frac{dtradia \times sum_{old}}{zicp \times soilcap + dtradia \times sum\_sns}\left(unit{:}\ \frac{s\times\frac{W}{m^2}}{s\times\frac{kg}{m^2\times s}} = \frac{W}{m^2}\times\frac{m^2\times s}{kg} = \frac{J}{kg}\right)$$

$$dtradia = \delta t$$

$$unit{:}\ \frac{soilcap}{cp\_air} = \frac{J}{m^2\times K}\times\frac{kg\times K}{J} = \frac{kg}{m^2}, dtradia\times sum_{sns} = s\times\frac{kg}{m^2\times s} = \frac{kg}{m^2}$$

(3) Surface temperature:

Finally, by using E.Q. (64) and E.Q. (70) to replace F_1^H and F_1^q in E.Q. (A29), we obtain T_S^t. ('temp_sol_new' in source code)

$$\Rightarrow Cs\frac{T_S^t}{\delta t} + 4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3 T_S^t - N_1^h T_S^t - LN_1^q T_S^t$$

$$= Fs + Rsw_{net} + Rlw_{down} - \left[\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 - 4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + (1-\varepsilon)\times Rlw_{down}\right] + M_1^h$$

$$+ LM_1^q + Cs\frac{T_S^{t-\delta t}}{\delta t}$$

$$\Rightarrow T_S^t = \frac{Fs + Rsw_{net} + Rlw_{down} - \left[-3\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + (1-\varepsilon)\times Rlw_{down}\right] + M_1^h + LM_1^q + Cs\frac{T_S^{t-\delta t}}{\delta t}}{\frac{Cs}{\delta t} + 4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3 - N_1^h - LN_1^q}$$

$$= \frac{CsT_S^{t-\delta t} + \delta t\left\{Fs + Rsw_{net} + Rlw_{down} - \left[-3\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + (1-\varepsilon)\times Rlw_{down}\right] + M_1^h + LM_1^q\right\}}{Cs + \delta t\left[4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3 - \left(N_1^h + LN_1^q\right)\right]}$$

$$= \frac{CsT_S^{t-\delta t} + \delta t\left\{Fs + Rsw_{net} + \left[\varepsilon Rlw_{down} - \varepsilon\sigma\left(T_S^{t-\delta t}\right)^4\right] + 4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + M_1^h + LM_1^q\right\}}{Cs + \delta t\left[4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3 - \left(N_1^h + LN_1^q\right)\right]}$$

In SUBROUTINE enerbil_surftemp:

$$psold = C_p T_s = temp\_sol \times cp\_air\left(unit{:}\ \frac{J}{kg\times K}\times K = \frac{J}{kg}\right)$$

$$\text{psnew} = \text{psold} + \text{dtheta}$$

$$\text{temp}_{\text{sol}_{\text{new}}} = \frac{\text{psnew}}{\text{cp}_{\text{air}}} = \frac{\text{psold} + \text{dtheta}}{\text{cp}_{\text{air}}} = \frac{\text{temp}_{\text{sol}} \times \text{cp}_{\text{air}} + \frac{\text{dtradia} \times \text{sum}_{\text{old}}}{\text{zicp} \times \text{soilcap} + \text{dtradia} \times \text{sum}_{\text{sns}}}}{\text{cp}_{\text{air}}}$$

$$= \text{temp}_{\text{sol}} + \frac{\text{dtradia} \times \text{sum}_{\text{old}}}{\text{cp}_{\text{air}} \times \text{dtradia} \times \text{sum}_{\text{sns}} + \text{soilcap}}$$

$$= \frac{\text{temp}_{\text{sol}} \times \text{soilcap} + \text{temp}_{\text{sol}} \times (\text{cp}_{\text{air}} \times \text{dtradia} \times \text{sum}_{\text{sns}}) + \text{dtradia} \times \text{sum}_{\text{old}}}{\text{cp}_{\text{air}} \times \text{dtradia} \times \text{sum}_{\text{sns}} + \text{soilcap}}$$

$$= \frac{\text{temp}_{\text{sol}} \times \text{soilcap} + \text{dtradia} \times [\text{temp}_{\text{sol}} \times (\text{cp}_{\text{air}} \times \text{sum}_{\text{sns}}) + \text{sum}_{\text{old}}]}{\text{cp}_{\text{air}} \times \text{dtradia} \times \text{sum}_{\text{sns}} + \text{soilcap}}$$

$$= \frac{\text{temp}_{\text{sol}} \times \text{soilcap} + \text{dtradia} \times \left\{\text{temp}_{\text{sol}} \times \left[-N_1^h - LN_1^q + 4\varepsilon\sigma\left(T_S^{t-\delta t}\right)^3\right] + \text{sum}_{\text{old}}\right\}}{\text{soilcap} + \text{dtradia} \times \text{cp}_{\text{air}} \times \text{sum}_{\text{sns}}}$$

$$= \frac{\text{temp}_{\text{sol}} \times \text{soilcap} + \text{dtradia} \times \left[\text{Fs} + \text{Rsw}_{\text{net}} + \varepsilon\text{Rlw}_{\text{down}} + 3\varepsilon\sigma\left(T_S^{t-\delta t}\right)^4 + M_1^h + LM_1^q\right]}{\text{soilcap} + \text{dtradia} \times \text{cp}_{\text{air}} \times \text{sum}_{\text{sns}}}$$

4.4.7. The calculation of water vapor flux and enthalpy

Once the surface temperature Ts is known, the water vapor flux and enthalpy flux are immediately calculated from E.Q. (64) and E.Q. (70).

(1) Calculation of epot_air_new, tair qair_new, fevap:

Calculation of $\text{epot}_{\text{air}_{\text{new}}}$, e.q. (62) in Dufresne and Ghattas, (2009):

$$F_1^H = K_1^H \left[H_1 - C_p \left(\frac{P_r}{P}\right)^{\kappa} T_S\right] (62) \Rightarrow H_1 = \frac{F_1^H}{K_1^H} + C_p \left(\frac{P_r}{P}\right)^{\kappa} T_S \overset{\frac{P_r}{P}\approx 1}{\Longrightarrow} H_1 = \frac{1}{K_1^H} \times F_1^H + C_p T_S$$

In code:

$$\text{epot}_{\text{air}_{\text{new}}} = \text{zikt} \times (\text{sensfl}_{\text{old}} - \text{sensfl}_{\text{sns}} \times \text{dtheta}) + \text{psnew} \left(\text{unit: } \frac{J}{kg}\right)$$

$$\text{tair} = \frac{\text{epot\_air}}{\text{cp\_air}} = \frac{\text{zikt} \times (\text{sensfl}_{\text{old}} - \text{sensfl}_{\text{sns}} \times \text{dtheta}) + \text{psnew}}{\text{cp\_air}} = \frac{\text{zikt} \times \frac{(\text{petBcoef} - \text{cp}_{\text{air}} \times T_S^t)}{\text{zikt} - \text{petAcoef}} + \text{psnew}}{\text{cp\_air}}$$

Calculation of qair_{new}, e.q. (67) in Dufresne and Ghattas, (2009):

$$F_1^q = \beta K_1^q (Q - q_0) (67) \Rightarrow Q = \frac{F_1^q}{\beta K_1^q} + q_0$$

In code, SUBROUTINE enerbil_flux:

$$\text{qair}_{\text{new}} = \text{zikq} \times \frac{1}{\text{chalsu0} \times \text{vbeta1} \times (1 - \text{vbeta5}) + \text{chalev0} \times [(1 - \text{vbeta1}) \times (1 - \text{vbeta5}) \times \text{vbeta} \times \text{valpha} + \text{vbeta5}]} \times \text{fevap} + \text{qsol}_{\text{sat}_{\text{new}}} \left(\text{unit:} \frac{m^3}{kg} \times \frac{s}{m} \times \frac{kg}{J} \times \frac{W}{m^2} = \frac{kg}{kg}\right)$$

$$\text{fevap} = (\text{lareva\_old} - \text{lareva\_sns} \times \text{dtheta}) + (\text{larsub\_old} - \text{larsub\_sns} \times \text{dtheta}) \left(\text{unit:} \frac{W}{m^2}\right)$$

(2) To calculate fluxsens and fluxlat:

According equation e.q. (62) in Dufresne and Ghattas, (2009):

$$\text{fluxsens} = \frac{\text{cp}_{\text{air}} \times \text{temp\_sol\_new} - \text{petBcoef}}{\text{zikt} - \text{petAcoef}}$$

in code: SUBROUTINE enerbil_flux

$$\mathrm{fluxsens} = \mathrm{rau} \times \mathrm{speed} \times \mathrm{q_{cdrag}} \times (\mathrm{psnew} - \mathrm{epot_{air}})$$

According equation e.q. (67) in Dufresne and Ghattas, (2009)

$$\mathrm{fluxlat} = \{\mathrm{chalsu0} \times \mathrm{vbeta1} \times (1 - \mathrm{vbeta5}) + \mathrm{chalev0} \times [(1 - \mathrm{vbeta1}) \times (1 - \mathrm{vbeta5}) \times \mathrm{vbeta} + \mathrm{vbeta5}]\} \times \frac{\mathrm{qsol_{sat_{new}}} - \mathrm{peqBcoef}}{\mathrm{zikq} - \mathrm{peqAcoef}}$$

in code: SUBROUTINE enerbil_flux

$$\mathrm{fluxlat} = \mathrm{chalsu0} \times \mathrm{rau} \times \mathrm{qc} \times \mathrm{vbeta1} \times (1 - \mathrm{vbeta5}) \times \left(\mathrm{qsol_{sat_{new}}} - \mathrm{qair}\right) + \mathrm{chalev0} \times \mathrm{rau} \times \mathrm{qc} \times \mathrm{vbeta5} \times \left(\mathrm{qsol_{sat_{new}}} - \mathrm{qair}\right) + \mathrm{chalev0} \times \mathrm{rau} \times \mathrm{qc} \times (1 - \mathrm{vbeta1}) \times (1 - \mathrm{vbeta5}) \times \mathrm{vbeta} \times \left(\mathrm{valpha} \times \mathrm{qsol_{sat_{new}}} - \mathrm{qair}\right)$$

$$\mathrm{vevapp} = \mathrm{dtradia} \times \mathrm{rau} \times \mathrm{qc} \times [\mathrm{vbeta1} \times (1 - \mathrm{vbeta5}) + \mathrm{vbeta5}] \times \left(\mathrm{qsol_{sat_{new}}} - \mathrm{qair\_new}\right) + \mathrm{dtradia} \times \mathrm{rau} \times \mathrm{qc} \times [(1 - \mathrm{vbeta1}) \times (1 - \mathrm{vbeta5}) \times \mathrm{vbeta}] \times \left(\mathrm{valpha} \times \mathrm{qsol_{sat_{new}}} - \mathrm{qair\_new}\right)$$

A new subroutine ‘SUBROUTINE enerbil_surftempflux’ was written by merging SUBROUTINE ‘enerbil_surftemp’ and ‘enerbil_flux’.

Some constants in the code

Ratio between specific constant and specific heat of dry air:

$$\mathrm{kappa} = \frac{\mathrm{cte\_molr}}{\mathrm{cp\_air}} = \frac{287.05(\mathrm{kg/mol})}{1004.675\left(\frac{\mathrm{J}}{\mathrm{kg}}/\mathrm{K}\right)} \approx 0.2857\left(\mathrm{kg/mol} \times \frac{\mathrm{kg} \times \mathrm{K}}{\mathrm{J}}\right)$$

Specific heat of dry air:

$$\mathrm{cp_{air}} = 1004.675\left(\frac{\mathrm{J}}{\mathrm{kg} \times \mathrm{K}}\right)$$

Stefan-boltzman constant:

$$\mathrm{c_{stefan}} = 5.6697 \times 10^{-8}\left(\mathrm{unit:}\ \frac{\mathrm{W}}{\mathrm{m}^2 \times \mathrm{K}^4}\right)$$

Latent heat of sublimation:

$$\mathrm{chalsu0} = 2.8345 \times 10^6 (\mathrm{J/kg})$$

Latent heat of evaporation:

$$\mathrm{chalev0} = 2.5008 \times 10^6 (\mathrm{J/kg})$$

References:

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Dufresne, J-L & Ghattas, J, 2009. Description du schema de la couche limite turbulente et la interface avec la surface planetaire dans LMDZ, Technical note, available (22/12/11): http://lmdz.lmd.jussieu.fr/developpeurs/notes-techniques/ressources/pbl_surface.pdf