LEGOS
(laboratoire d’études en géophysique et océanographie spatiales)

CMEMS project « Tide – NEMO »

Implementation of the barotropic tide in the NEMO model

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NEMO-TIDE-WG 12/01/2021

HYPOTHESIS (for the methodology to work)

Tide = forced phenomenon predictable by an “inexpensive” model (barotropic)

Accuracy depends on the spatial resolution, bathymetry, SAL, wave drag and optimization of the bottom friction coefficients (possibly by data assimilation)

There is an ideal (= reference/target) solution given by a simplified model (e.g. FES) optimized for tides

! for lack of adapted resolution and optimization based on tides, an OGCM will not give a tide as good as this reference model
! We therefore seek to get as close as possible to the reference solution

⇒ Proposed methodology allows to reproduce the ideal tide model solution projected on the OGCM grid.
⇒ Here Tide model = FES and OGCM = NEMO

General status of our current work

- Methodology defined
- Implementation in NEMO « done »

⇒ New routine init_tideFES_dynspg_ts : based on dynspg_ts
Calculates fields (projected FES solution UFES => SSH_FES and forcing fields for momentum)

⇒ Modification of dynsp_ts to use the new forcing

⇒ Other subroutines (where tide signal is used) have to be modified too but marginally…

- Tests Configuration « ORCA025 » (1/4°) with « academic FES »

- Tests Configuration « ORCA025 » (1/4°) with « realistic FES »
Underway (interpollation of FES solutions on OGCM grid has to be carefully done)

METHODOLOGY

Consider 1 mode (solution = linear superimposition of modes)

$$
\begin{aligned}
\partial_t \overline{\mathbf{U}}^{TM} + \mathbf{f} \times \overline{\mathbf{U}}^{TM} &= -g\,\nabla \zeta^{TM} + \nabla \Phi_{astro} + \mathbf{X}^{TM}, \\
\partial_t \zeta^{TM} + div(H\,\overline{\mathbf{U}}^{TM}) &= 0.
\end{aligned}
$$

Tide model

$$
\begin{aligned}
\partial_t \overline{\mathbf{U}} + \mathbf{f} \times \overline{\mathbf{U}} &= -g\,\nabla \zeta + \nabla \Phi_{astro} + \overline{\partial_t \mathbf{U}^*} + \mathbf{X}, \\
\partial_t \zeta + div((H+\zeta)\,\overline{\mathbf{U}}) &= 0,
\end{aligned}
$$

OGCM, barotropic Eq.

Similar EDP but different implementations/discretization
$\Rightarrow$ Different solutions because of different schemes
$\Rightarrow$ Correct scheme's weaknesses

METHODOLOGY

Solution we want to reach (=reference/target)

$(\overline{\mathbf{U}}^{OM},\ \zeta^{OM})$ **Projection of** $\overline{\mathbf{U}}^{TM}$**(**$\zeta^{TM}$**)** **on OGCM grid**

How must we modify the OGCM schemes to get $(\overline{\mathbf{U}}^{OM},\ \zeta^{OM})$ **?**

$$
\begin{aligned}
\partial_t \overline{\mathbf{U}} + \mathbf{f} \times \overline{\mathbf{U}} &= -g\, \nabla\zeta + \nabla\Phi_{astro} + \overline{\partial_t \mathbf{U}^*} + \mathbf{X}, \\
\partial_t \zeta + div((H+\zeta)\, \overline{\mathbf{U}}) &= 0,
\end{aligned}
$$

METHODOLOGY

Solution we want to reach (=reference/target)

$(\overline{\mathbf{U}}^{OM},\ \zeta^{OM})$ **Projection of** $\overline{\mathbf{U}}^{TM}(\zeta^{TM})$ **on OGCM grid**

How must we modify the OGCM schemes to get $(\overline{\mathbf{U}}^{OM},\ \zeta^{OM})$ **?**

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\begin{aligned}
\partial_t \overline{\mathbf{U}} + \mathbf{f} \times \overline{\mathbf{U}} &= -g\,\nabla\zeta + \nabla\Phi_{astro} + \overline{\partial_t \mathbf{U}^*} + \mathbf{X}, \\
\partial_t \zeta + div((H + \cancel{\zeta})\,\overline{\mathbf{U}}) &= 0,
\end{aligned}
$$

linearize

gather these terms into 1 term

F^{OM}

METHODOLOGY

Solution we want to reach (=reference/target)

$(\overline{\mathbf{U}}^{OM},\ \zeta^{OM})$ **Projection of** $\overline{\mathbf{U}}^{TM}(\zeta^{TM})$ **on OGCM grid**

How must we modify the OGCM schemes to get $(\overline{\mathbf{U}}^{OM},\ \zeta^{OM})$ **?**

ζ^{OM} and F^{OM} are « readily » determined …

$$
\begin{aligned}
\partial_t \overline{\mathbf{U}}^{OM} + \mathrm{f} \times \overline{\mathbf{U}}^{OM} - g\,\nabla\zeta^{OM} &= \mathbf{F}^{OM}, \\
\partial_t \zeta^{OM} + div(H\,\overline{\mathbf{U}}^{OM}) &= 0.
\end{aligned}
$$

METHODOLOGY

Solution we want to reach (=reference/target)

$(\overline{\mathbf{U}}^{OM},\ \zeta^{OM})$ **Projection of** $\overline{\mathbf{U}}^{TM}(\zeta^{TM})$ **on OGCM grid**

How must we modify the OGCM schemes to get $(\overline{\mathbf{U}}^{OM},\ \zeta^{OM})$ **?**

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\begin{aligned}
\partial_t \overline{\mathbf{U}}^{OM} + \mathbf{f} \times \overline{\mathbf{U}}^{OM} - g\, \nabla \zeta^{OM} &= \mathbf{F}^{OM}, \\
\partial_t \zeta^{OM} + div(H\, \overline{\mathbf{U}}^{OM}) &= 0.
\end{aligned}
$$

IMPORTANT
Discretized Operators

$\Rightarrow$ ζ^{OM} &F^{OM} adapted to numerical schemes

METHODOLOGY

In practice

Determine the (projected) TRANSPORT and split into cos/sin

$$(H\overline{\mathbf{U}})^{OM}(x,\ y,\ t) \ = \ (H\overline{\mathbf{U}})_c^{OM}(x,\ y)\ cos\ \omega t + (H\overline{\mathbf{U}})_s^{OM}(x,\ y)\ sin\ \omega t$$

OGCM SSH Eq

$$\zeta_{N+1} \ = \ Op_t[\zeta_N,\ \zeta_{N-1},\ \zeta_{N-2}] + Op_{x/y}[(H\overline{\mathbf{U}})_{N+1},\ (H\overline{\mathbf{U}})_N,\ (H\overline{\mathbf{U}})_{N-1},\ (H\overline{\mathbf{U}})_{N-2}] \tag{7}$$

$Op_t[.]$ and $Op_{x/y}[.]$ are discretized time and spatial operator

$$\zeta^{OM}(x,\ y,\ t) \ = \ \zeta_c^{OM}(x,\ y)\ cos\ \omega t + \zeta_s^{OM}(x,\ y)\ sin\ \omega t$$

METHODOLOGY

In practice

$$\begin{bmatrix} \alpha_c & \alpha_s \\ \beta_c & \beta_s \end{bmatrix} \times \begin{pmatrix} \zeta_c^{OM} \\ \zeta_s^{OM} \end{pmatrix} = \begin{pmatrix} Div_c^{tot} \\ Div_s^{tot} \end{pmatrix}$$

$$\begin{aligned}
\alpha_c &= cos(\omega\Delta t) - Op_t[1,\ cos(-\omega\Delta t),\ cos(-\omega 2\Delta t)] \\
\beta_c &= cos(\frac{\pi}{2} + \omega\Delta t) - Op_t[0,\ cos(\frac{\pi}{2} - \omega\Delta t),\ cos(\frac{\pi}{2} - \omega 2\Delta t)] \\
\alpha_s &= -\beta_c \\
\beta_s &= \alpha_c
\end{aligned}$$

$$\begin{aligned}
Div_c^{tot} &= Op_{x/y}[(H\overline{\mathbf{U}})_c,\ 0,\ 0,\ 0]\ cos(\omega\Delta t) \\
&+ Op_{x/y}[0,\ (H\overline{\mathbf{U}})_c,\ 0,\ 0]\ (1) \\
&+ Op_{x/y}[0,\ 0,\ (H\overline{\mathbf{U}})_c,\ 0]\ cos(\omega\Delta t) \\
&+ Op_{x/y}[0,\ 0,\ 0,\ (H\overline{\mathbf{U}})_c]\ co\ldots
\end{aligned}$$

$+ \ Op_{x/\ldots}[(H\ldots$ [illegible]

$+$ [illegible] $0]\ (0)$

$+$ [illegible] $_{x/y}[0,\ 0,\ (H\overline{\mathbf{U}})_s,\ 0]\ (-sin(\omega\Delta t))$

$+ \ Op_{x/y}[0,\ 0,\ 0,\ (H\overline{\mathbf{U}})_s]\ (-sin(2\omega\Delta t))$

$$Div_s^{tot} = Op_{x/y}[(H\overline{\mathbf{U}})_s,\ 0,\ 0,\ 0]\ cos(\ldots)$$

$+ \ Op_{x/y}[0,$ [illegible]

$+$ [illegible] $cos(\omega\Delta t)$

$+$ [illegible] $_{x/y}[0,\ 0,\ 0,\ (H\overline{\mathbf{U}})_s]\ cos(2\omega\Delta t)$

$$\begin{aligned}
&+ Op_{x/y}[(H\overline{\mathbf{U}})_c,\ 0,\ 0,\ 0]\ (-sin(\omega\Delta t)) \\
&+ Op_{x/y}[0,\ (H\overline{\mathbf{U}})_c,\ 0,\ 0]\ (0) \\
&+ Op_{x/y}[0,\ 0,\ (H\overline{\mathbf{U}})_c,\ 0]\ (sin(\omega\Delta t)) \\
&+ Op_{x/y}[0,\ 0,\ 0,\ (H\overline{\mathbf{U}})_c]\ (sin(2\omega\Delta t)).
\end{aligned}$$

Clumsy expressions, but solution determined from
A signle time step run of the barotropic scheme

METHODOLOGY

In practice

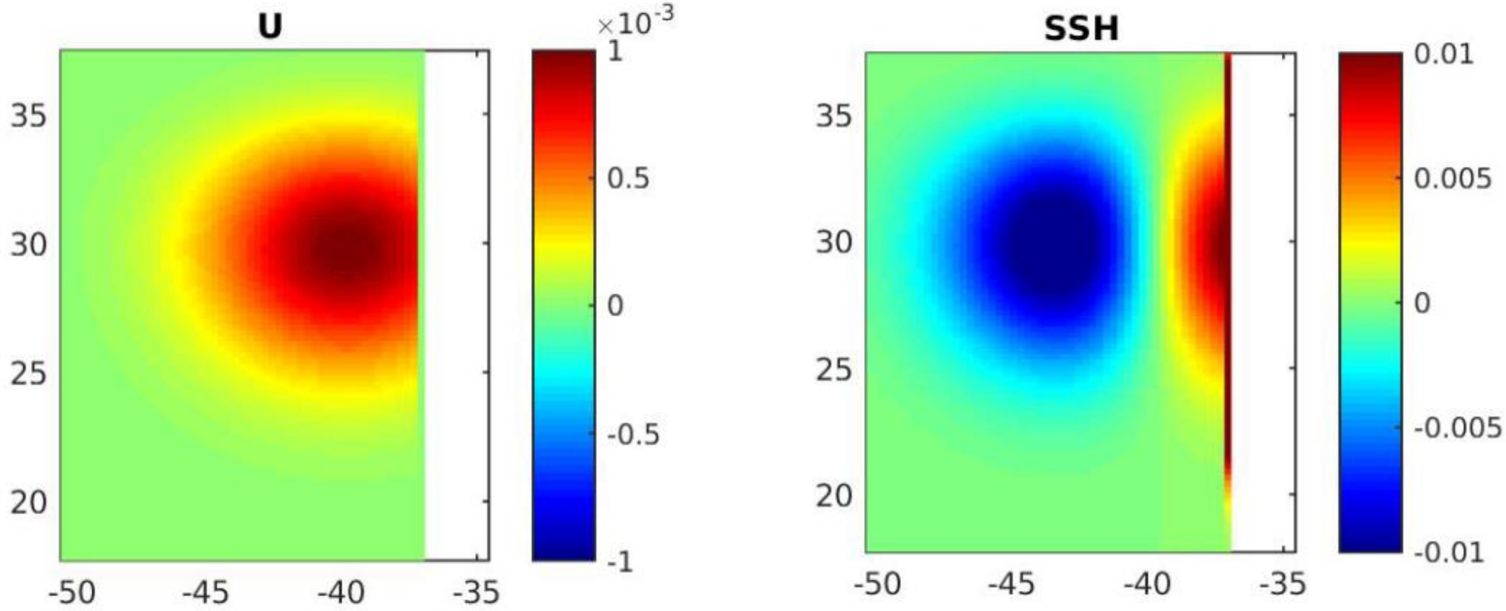
$$\mathrm{F}^{OM} \;=\; \overline{\mathbf{U}}^{OM}_{N+1} - Op_t[\overline{\mathbf{U}}^{OM}_{N},\ \overline{\mathbf{U}}^{OM}_{N-1},\ \overline{\mathbf{U}}^{OM}_{N-2}]$$

$$-Op_{x/y}[(\overline{\mathbf{U}}^{OM},\ \zeta^{OM})_{N+1},\ (\overline{\mathbf{U}}^{OM},\ \zeta^{OM})_{N},\ (\overline{\mathbf{U}}^{OM},\ \zeta^{OM})_{N-1},$$

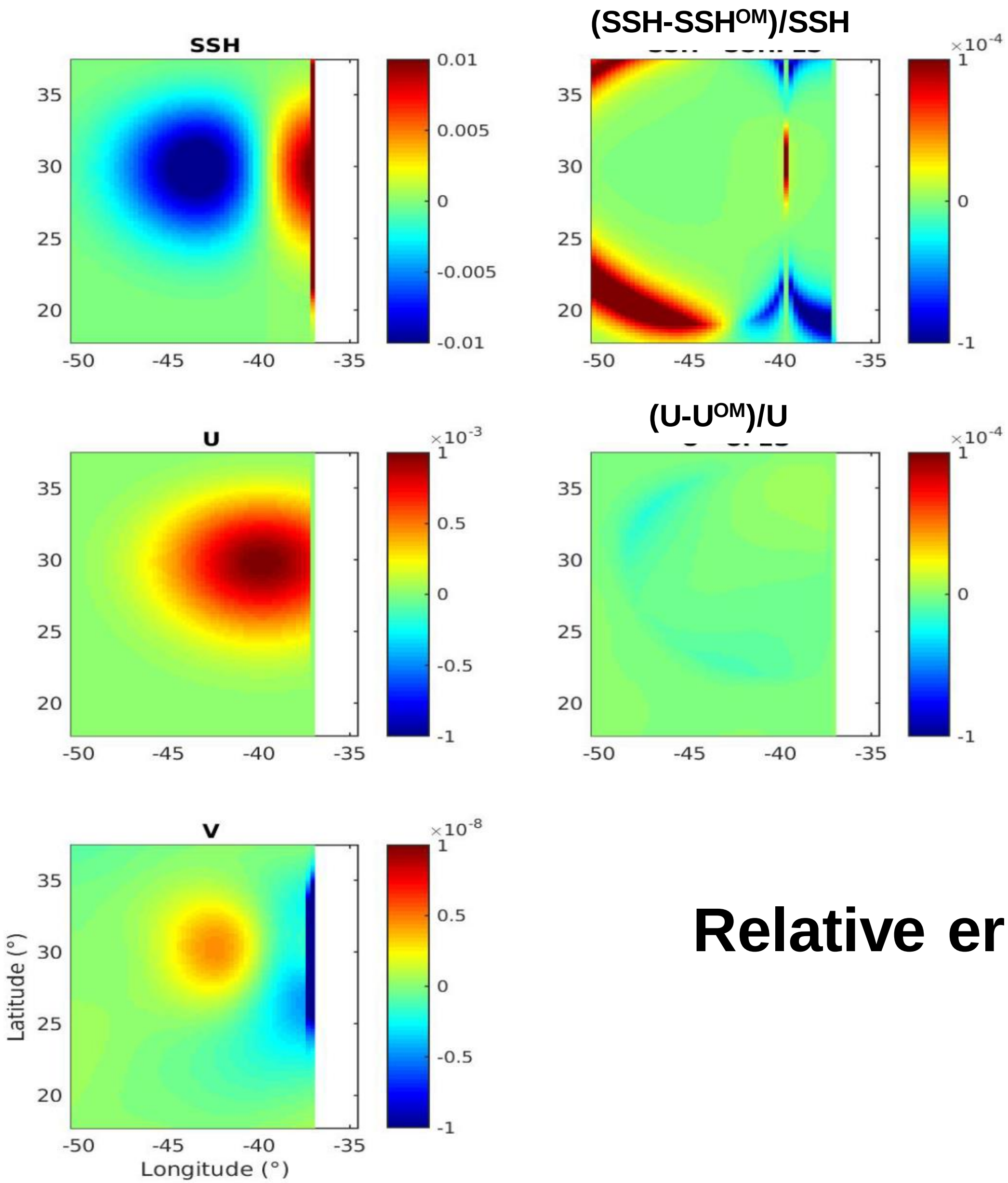
Similar approach to calculate the forcing terms …
(even more clumsy expressions but exact and adapted to chosen numerical schemes)

Results with academic fields

$$
\begin{aligned}
H\overline{U}_s^{OM} &= HU_0\ exp(-\frac{(x-x_0)^2+(y-y_0)^2}{R^2}), \\
H\overline{V}_s^{OM} &= 0, \\
H\overline{U}_c^{OM} &= 0, \\
H\overline{V}_c^{OM} &= 0
\end{aligned}
$$

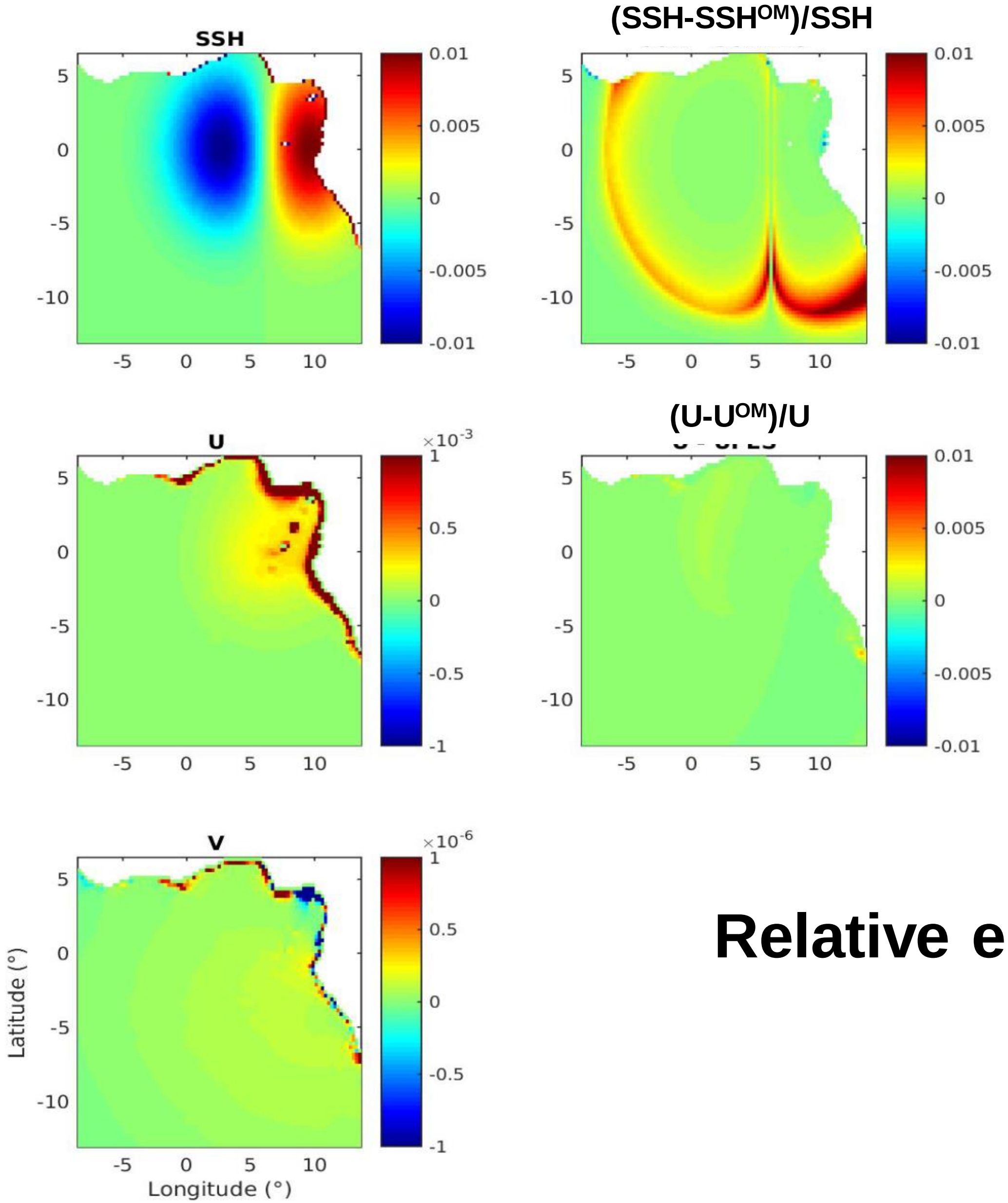


Results with academic fields



Relative error = very weak

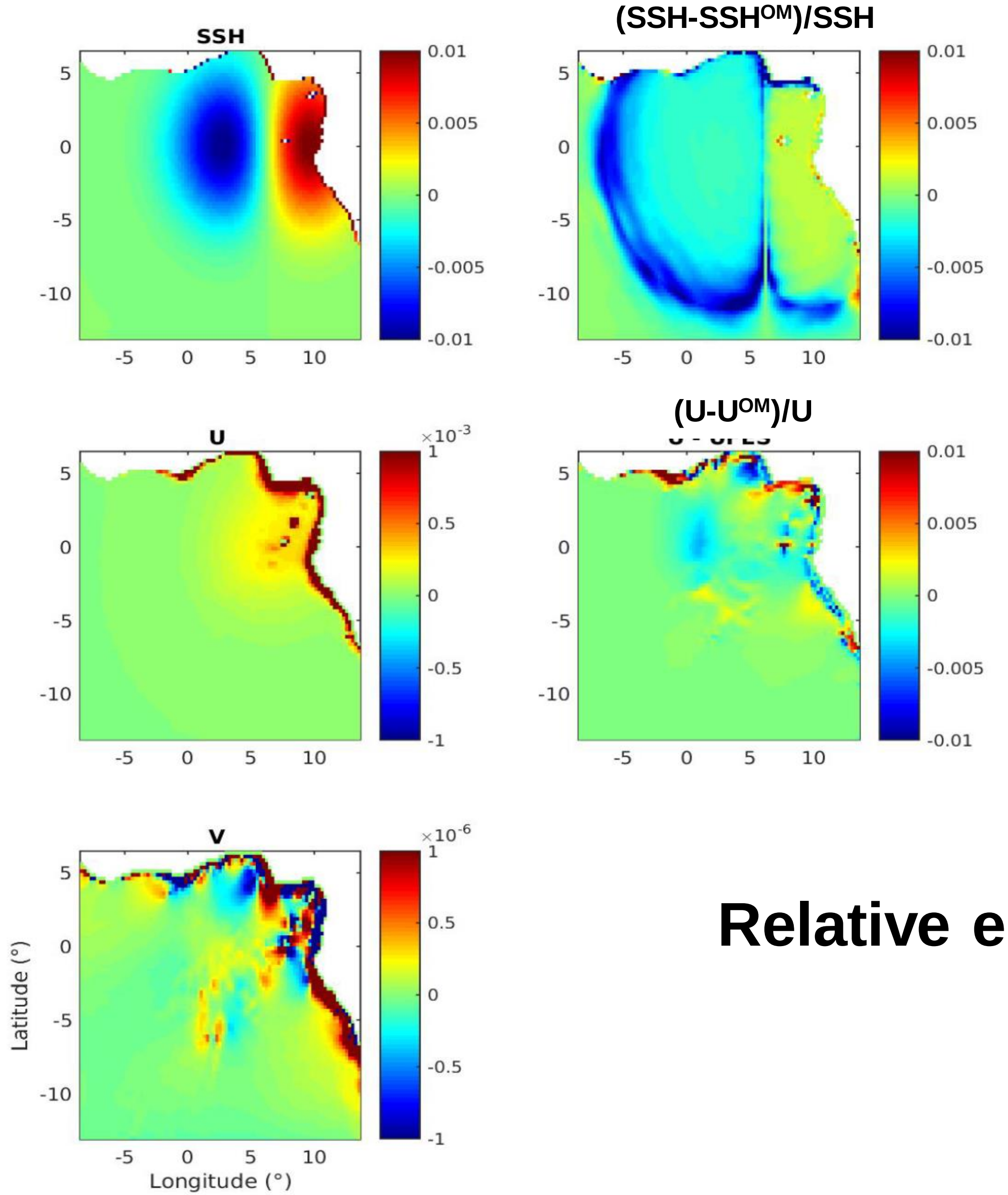
Results with academic fields



Academic +Realistic topography

Relative error = very weak

Results with academic fields



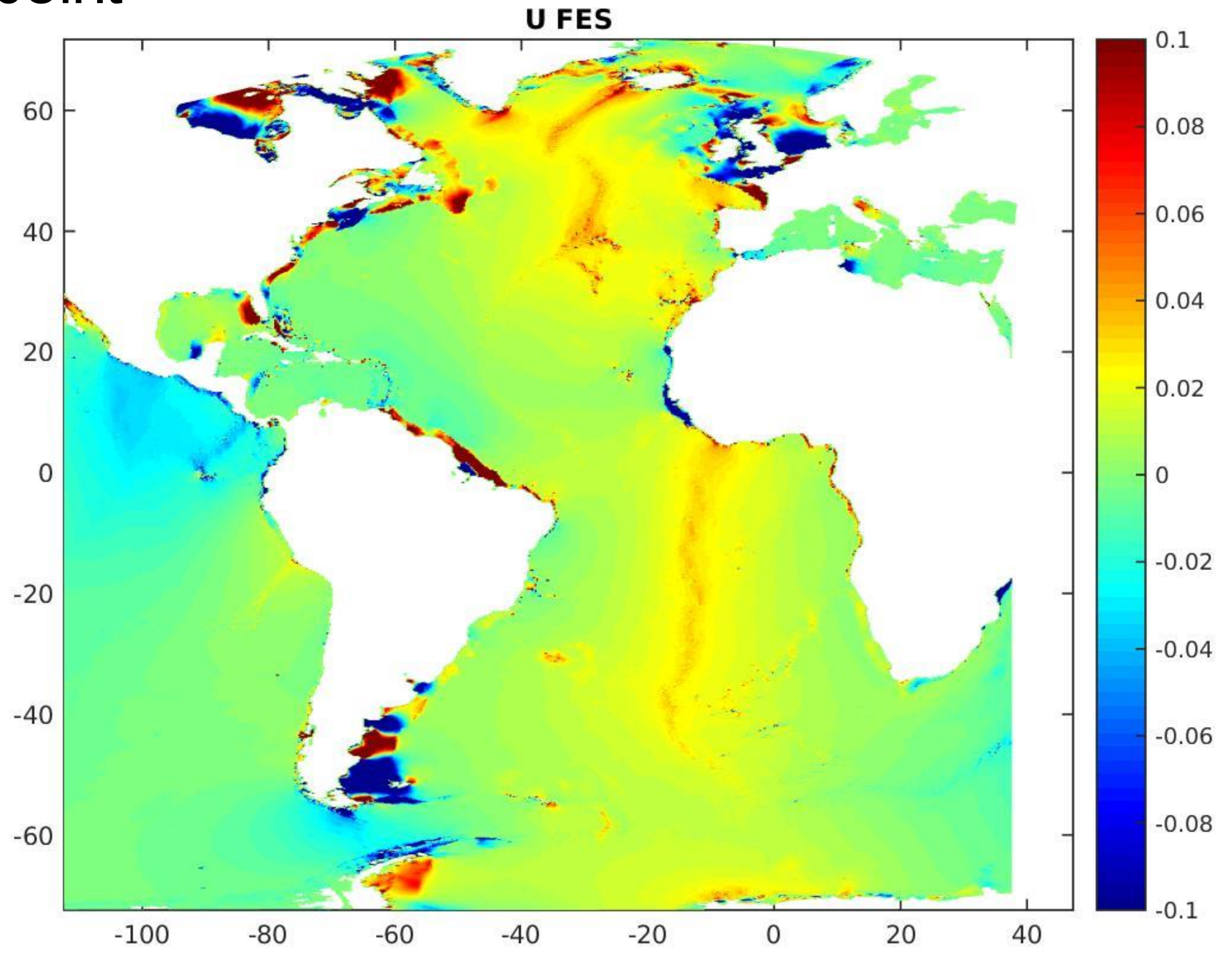
Academic
+Realistic topography
+nonlinear terms

Relative error = very weak

Results with real FES fields (M2)

FES Solution interpolated
On single NEMO grid point

U = Transport/Depth(NEMO)

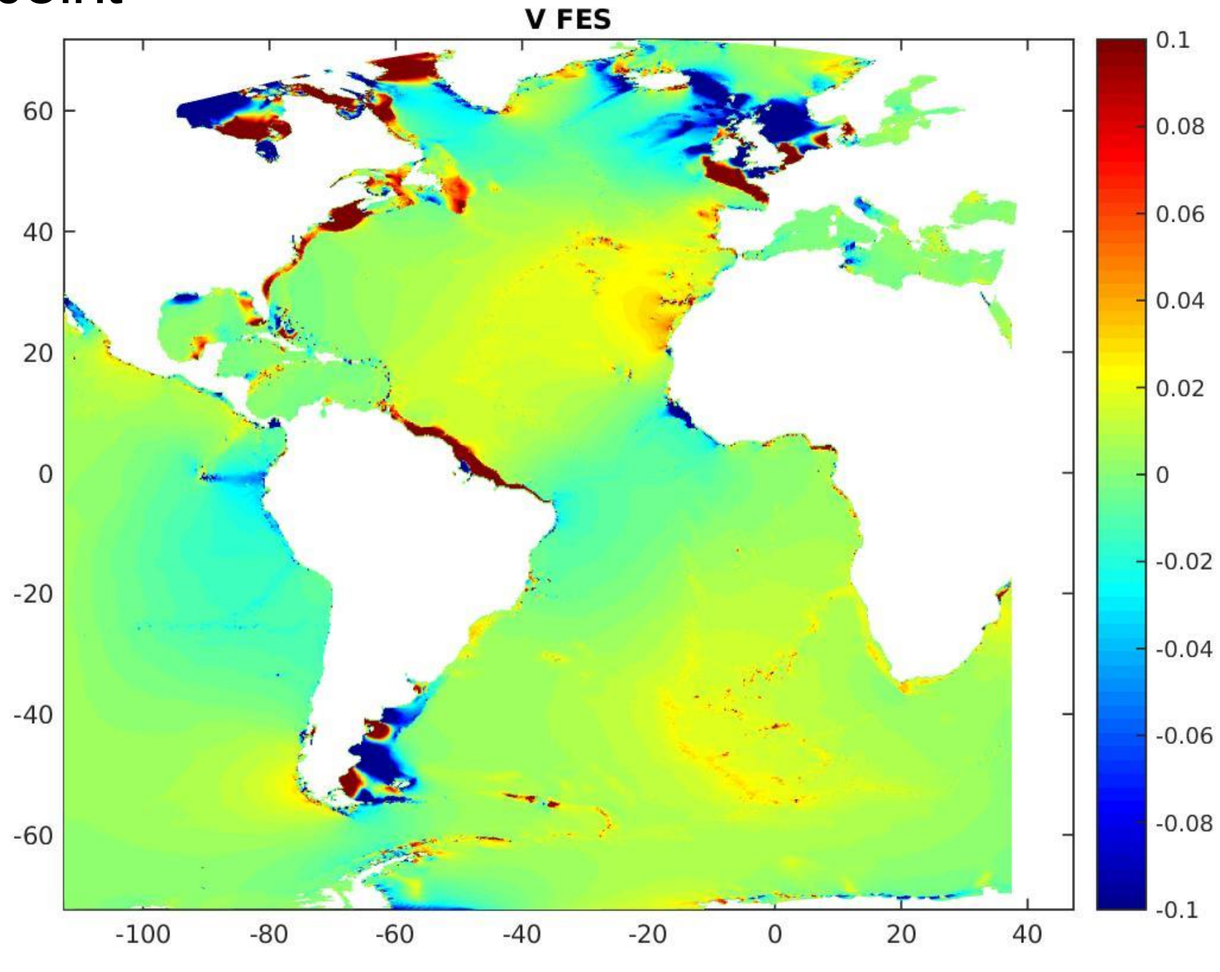


Current work

Results with real FES fields

FES Solution interpolated
On single NEMO grid point

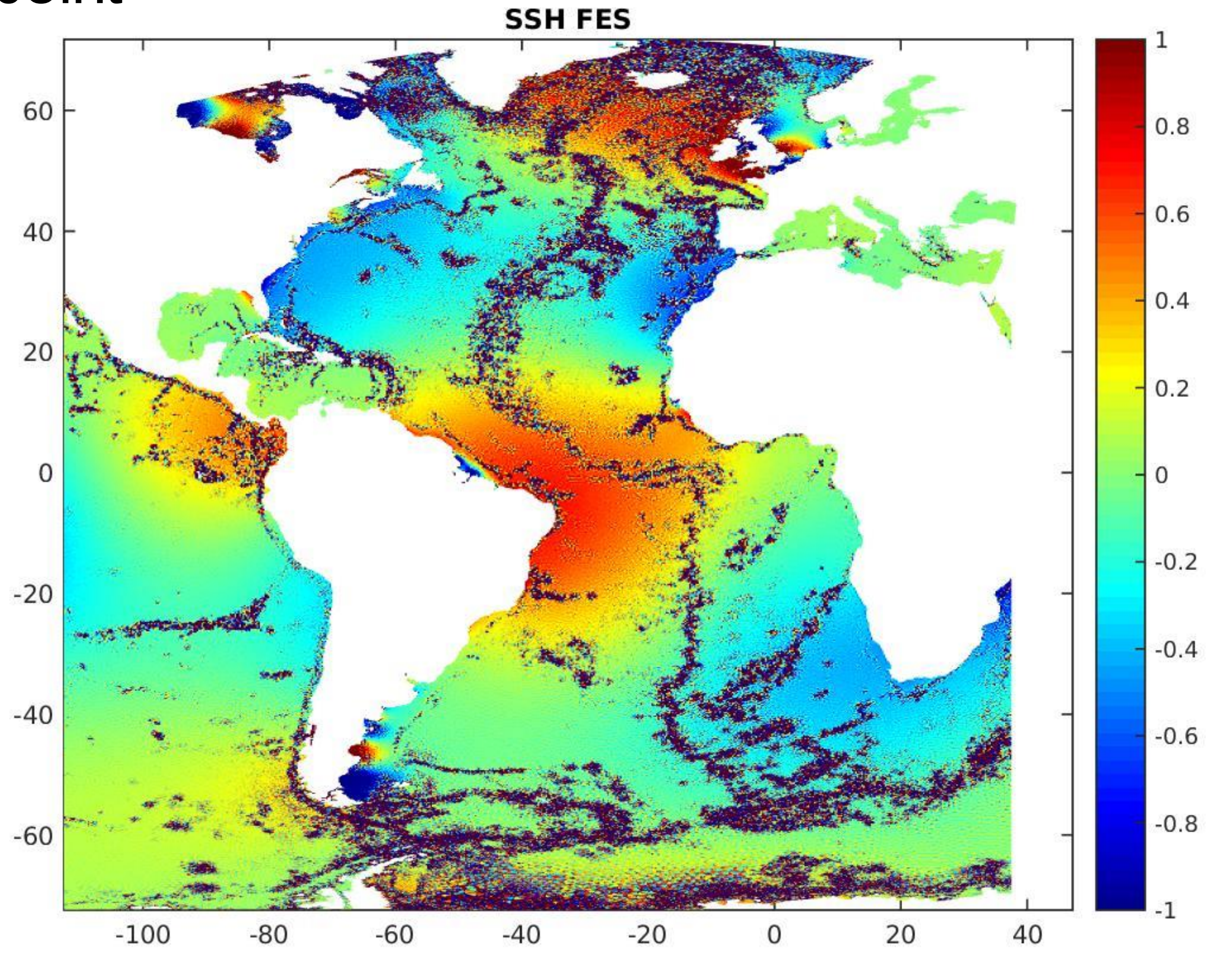
V = Transport/Depth(NEMO)



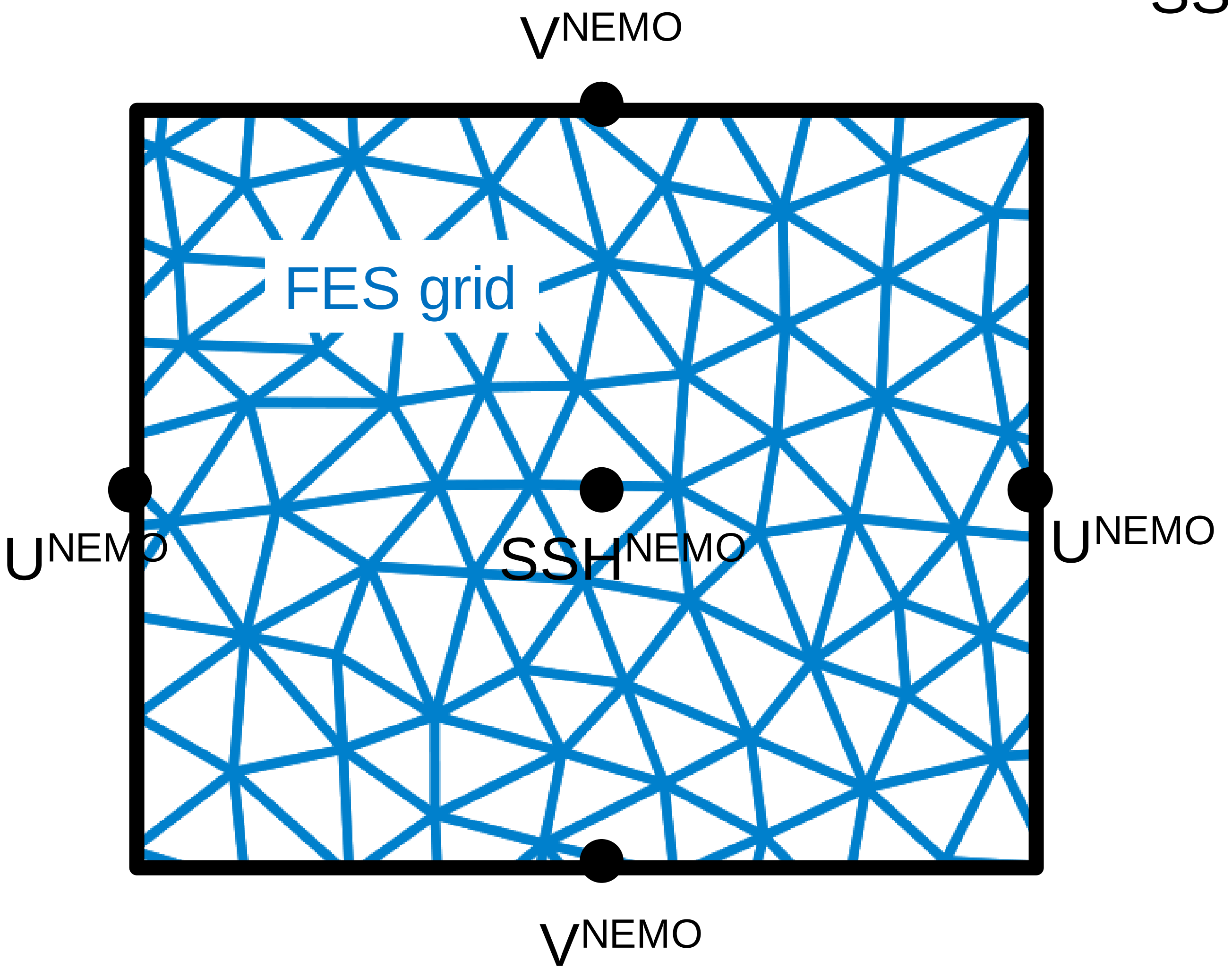
Results with real FES fields

FES Solution interpolated
On single NEMO grid point

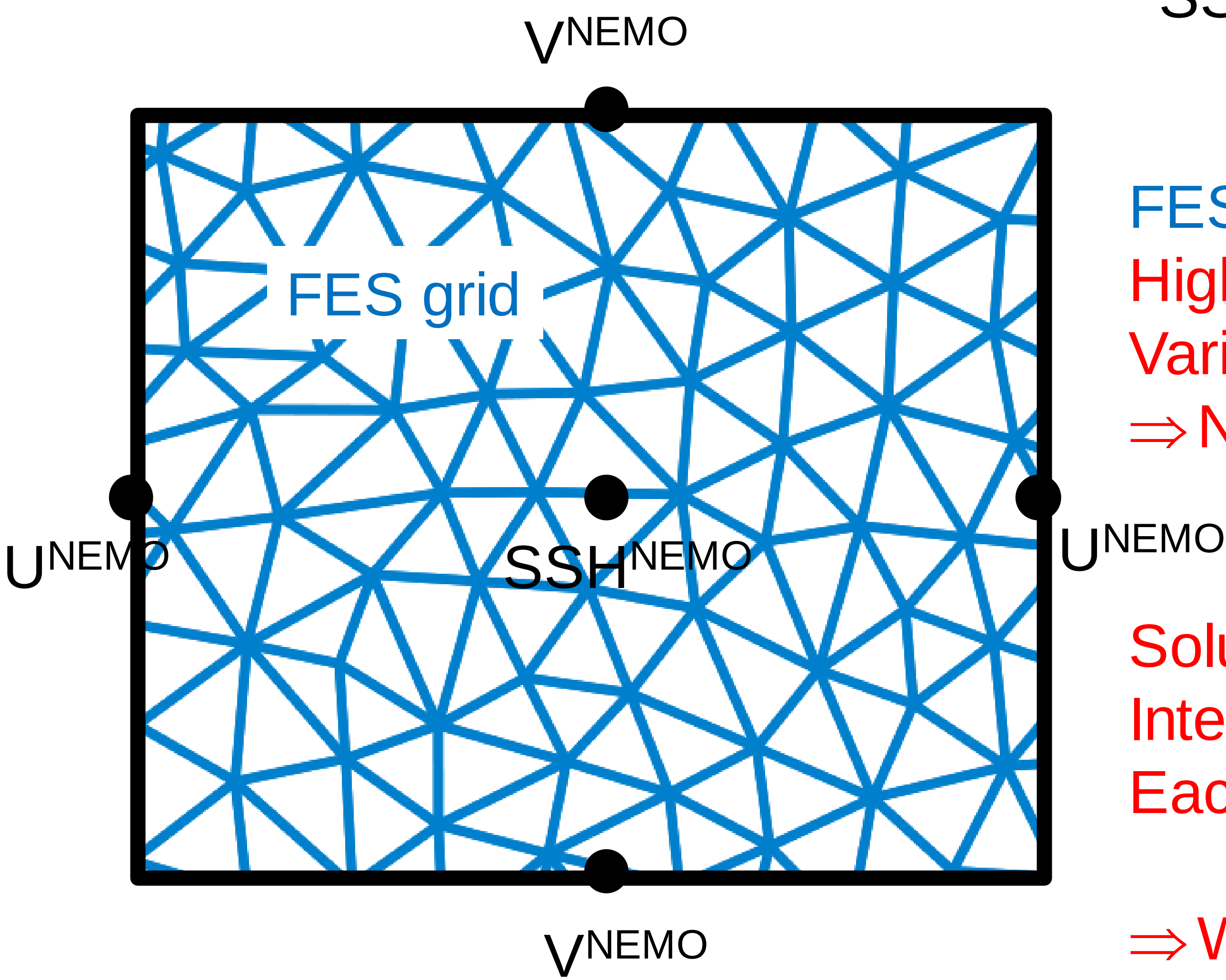
SSH = div (Transport)



$$SSH^{NEMO} = \mathrm{div}\ U^{NEMO}$$



$SSH^{NEMO} = \text{div } U^{NEMO}$



FES grid

High resolution / topo

Variability

$\Rightarrow$ Noise on NEMO grid

Solution :

Integrate FES fluxes along

Each NEMO grid cell side

$\Rightarrow$ Warrantees that

SSH^{NEMO}=mean(SSH^{FES})

CONCLUSION

Properties / advantages

The solution of the circulation model is the projected FES solution=reference

The cost is equivalent or even cheaper than a "normal" model (it is a simple coding of a forcing term which can be calculated once and for all for each harmonic component)

Time filtering is not required (to treat the bottom friction for instance)

Implementation of method validated (automatically adapts to num schemes)

Potential problems / disadvantage

-Implementation to finalize: currently difficulties with interpolation of FES solution on NEMO grid, but we think we know how to do …

-Other problems at poles (Achillee's heel of tide models)

- Non-linear term to look at (problem of rectification of the tide: a priori to be generated with the circulation model, but to be studied)
- If data assimilation used in FES (tide) model => equilibrium SSH=div(Transport) to be ensured …