



Evaluation of A Global Total Water Level Model in the Presence of Radiational S_2 Tide

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MOTIVATION

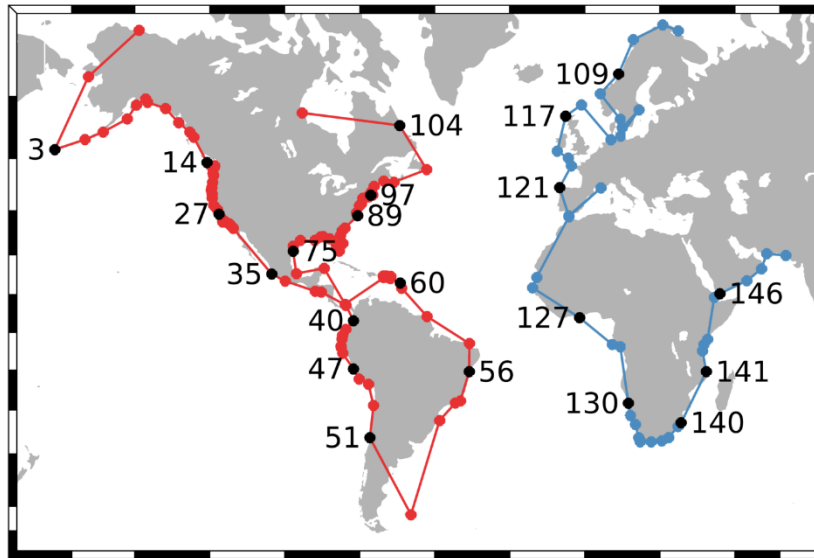
- Need for total water level (TWL) forecast for the benefits of Canadians
- Need for a global model
 - Allow enough room to resolve important coastal wave guides
 - Allow the inclusion of global processes (e.g., the oceanic response to atmospheric S_2 forcing over the tropics)
- Address the two following questions
 - How can we best predict tides using a model with limited spatial resolution? “Tidal nudging”?
 - What is the impact of neglecting nonlinear interactions on TWL prediction by a global model forced by hourly forcing?



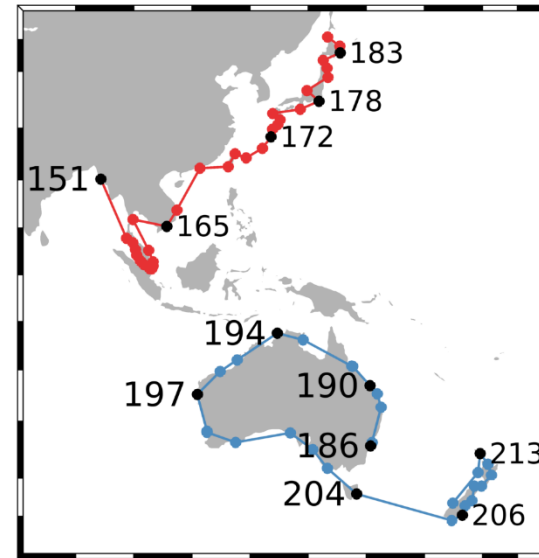
OBSERVATIONS

- TPXO8 (M2, S2, N2, K2, K1, O1, P1, and Q1)
- Tide gauge data from UHSLC in the year 2008

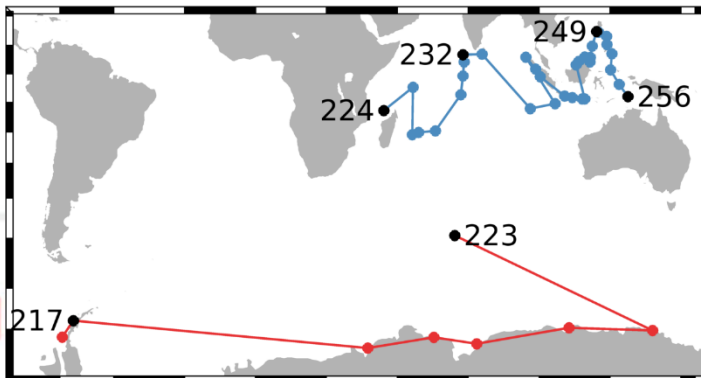
Am and EuA



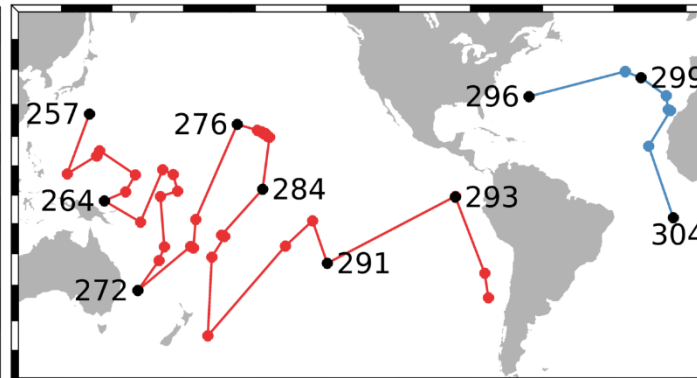
SEA and ANz



IO and Ant



Pis and Als



MODEL: GOVERNING EQUATIONS

Nudging \mathbf{u}

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{f} \times \mathbf{u} = -g \nabla \left((1 - \alpha_s) \eta - \eta_A \right) + A \nabla^2 \mathbf{u} + \frac{\tau_s - \tau_b}{\rho H} - \frac{1}{\rho} \nabla p_a - c_{iw} \mathbf{u} + \lambda(\mathbf{x}) \langle \mathbf{u}_{obs} - \mathbf{u} \rangle$$

Self-attraction and loading
Internal wave drag (This study)

$$\frac{\partial \eta}{\partial t} + \nabla \cdot (H \mathbf{u}) = 0$$

$$\mathbf{u}_{obs} = \frac{\text{Transport(TPX08)}}{\text{Depth(NEMO)}}$$

Nudging η

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{f} \times \mathbf{u} = -g \nabla \left((1 - \alpha_s) \eta - \eta_A \right) + A \nabla^2 \mathbf{u} + \frac{\tau_s - \tau_b}{\rho H} - \frac{1}{\rho} \nabla p_a - c_{iw} \mathbf{u}$$

$$\frac{\partial \eta}{\partial t} + \nabla \cdot (H \mathbf{u}) = \lambda(\mathbf{x}) \langle \eta_{obs} - \eta \rangle$$

(Kodaira et al., 2019)

Model setup based on NEMO

1/4°

Too coarse to predict tides and surges

1/12°

Acceptable tide and surge predictions on a global scale

1/36°

Only localized improvements at a considerable increase in cost

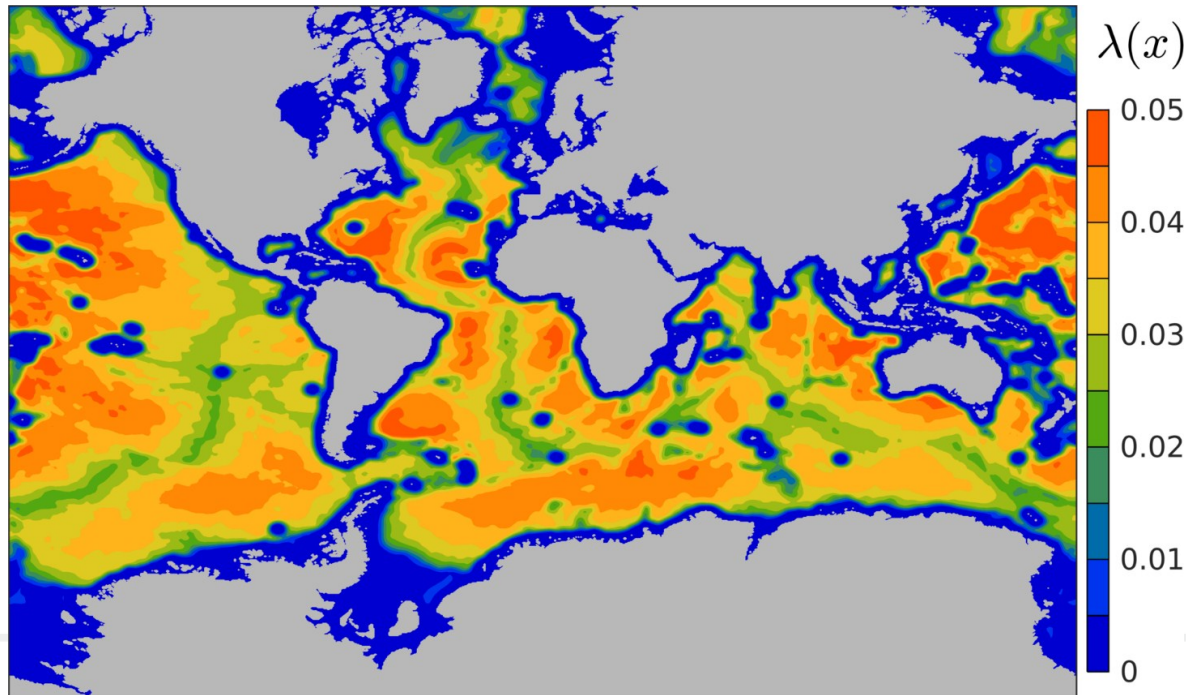
- Self-attraction and loading, internal wave drag (Kodaira et al., 2016)
- Surface wind stress formula (Bernier and Thompson, 2007)
- ORCA12 → eORCA12 grid: allow tidal propagation under ice shelves in the Ross Sea and Weddell Sea
- Tidal nudging



Spatial distribution of the nudging coefficient

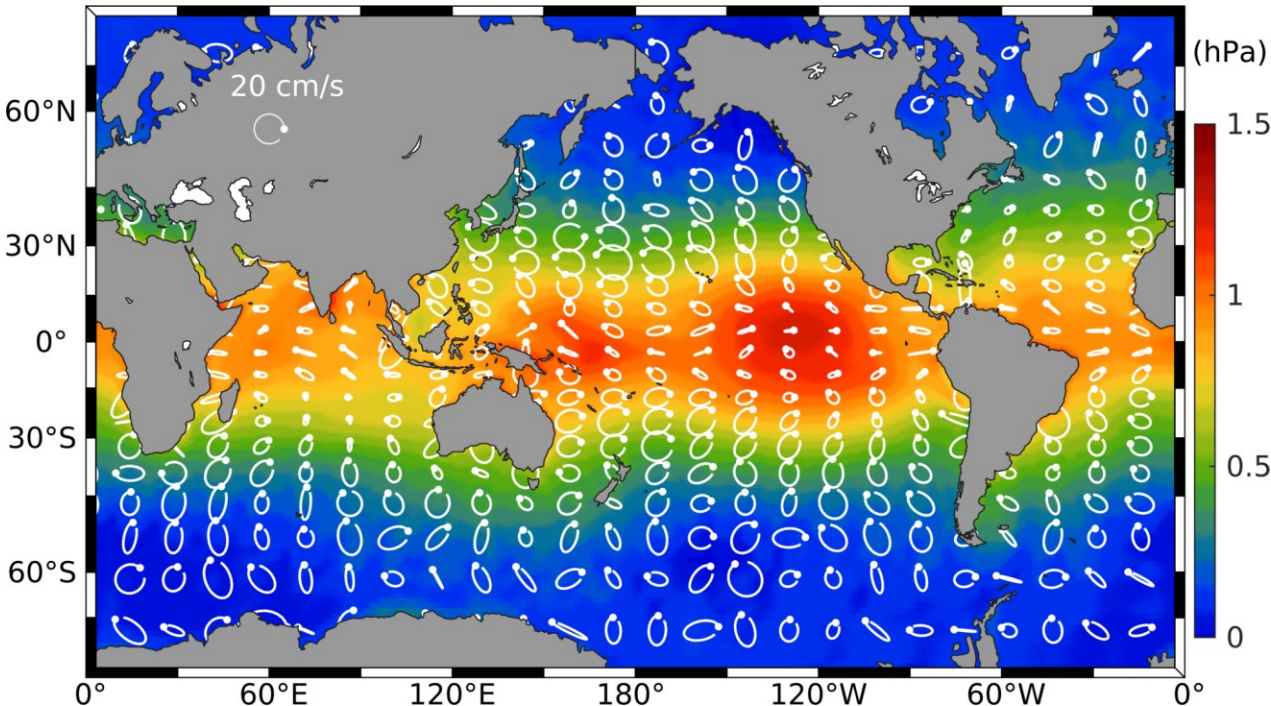
Nudge deep water only, allow surge and nonlinear processes to freely evolve on shelves

- On the shelf : $\lambda = 0$ for water depth shallower than ~ 400 m
- In deeper water: λ increases with water depth, spatially smoothed



ATMOSPHERIC FORCING (GDRS IN 2008, ~39 KM, HOURLY)

S_2 component of winds and air pressure



$$\tau_s = \tilde{\tau}_s + \tau'_s$$
$$p_a = \tilde{p}_a + p'_a$$

S_2 Residual

The hourly forcing has a significant S_2 tide which can trigger a global ocean response known as radiational S_2 tide (rS_2).



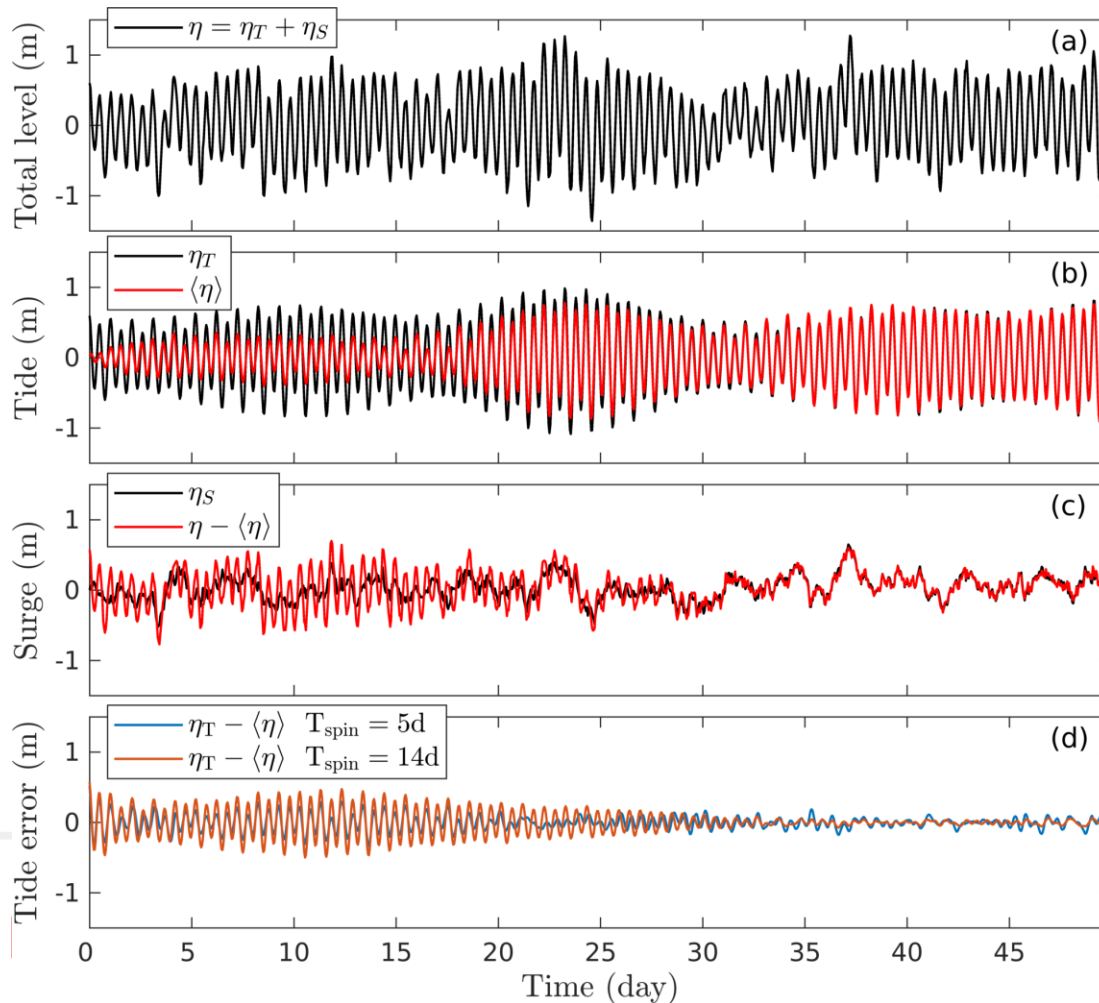
DESIGN OF EXPERIMENTS

(Tide-only, Surge-only, coupled tide-surge run)

| | Tidal potential | Full forcing | S ₂ signal removed | Nudging U | Nudging η |
|------------|---------------------|-----------------|-------------------------------|---------------------------------------------------------|---------------------------------------------|
| | η_A | (τ_s, p_a) | (τ'_s, p'_a) | $\lambda \langle \mathbf{u}_{obs} - \mathbf{u} \rangle$ | $\lambda \langle \eta_{obs} - \eta \rangle$ |
| Tide only | Run _T | ✓ | | | |
| | Run _{Tn} | ✓ | | ✓ | |
| | Run _{Tn} * | ✓ | | | ✓ |
| Surge only | Run _S | | ✓ | | |
| | Run _{S'} | | | ✓ | |
| Coupled | Run _{TS} | ✓ | ✓ | | |
| | Run _{TnS} | ✓ | ✓ | ✓ | |

Tidal Nudging (λ, κ)

- κ controls the width of the nudged bands and the spin-up time of the filter.
- Conceptually similar to applying a tidal analysis over a sliding window, and increasing κ is equivalent to reducing the window length.



η_S : surge (AR1 model)

η_T : tide (8 constituents)

$\langle \rangle$: tidal filter

$$T_{\text{spin}} = \kappa^{-1} \Delta t$$

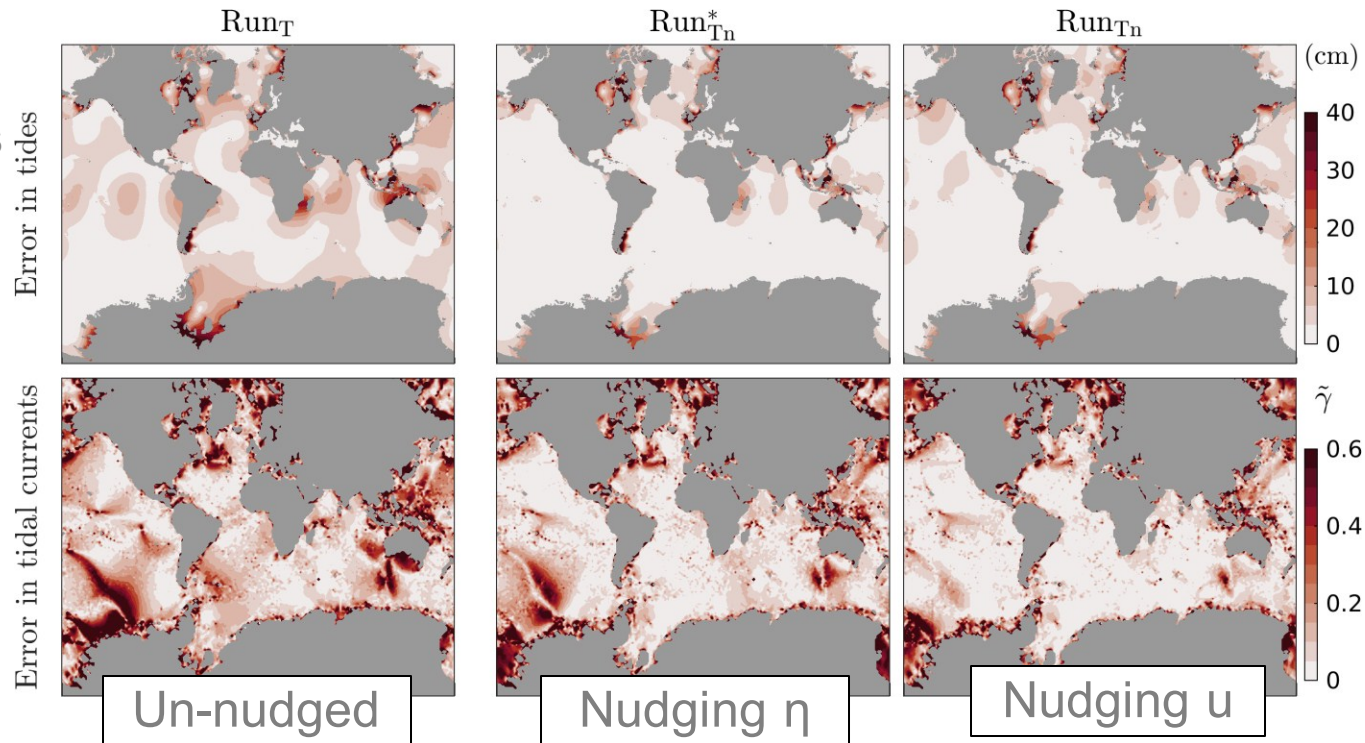
Nudging u VS. Nudging η


Comparison with TPXO8 for M2 tide (top) and tidal current (bottom)

Magnitude of differences in complex amplitude

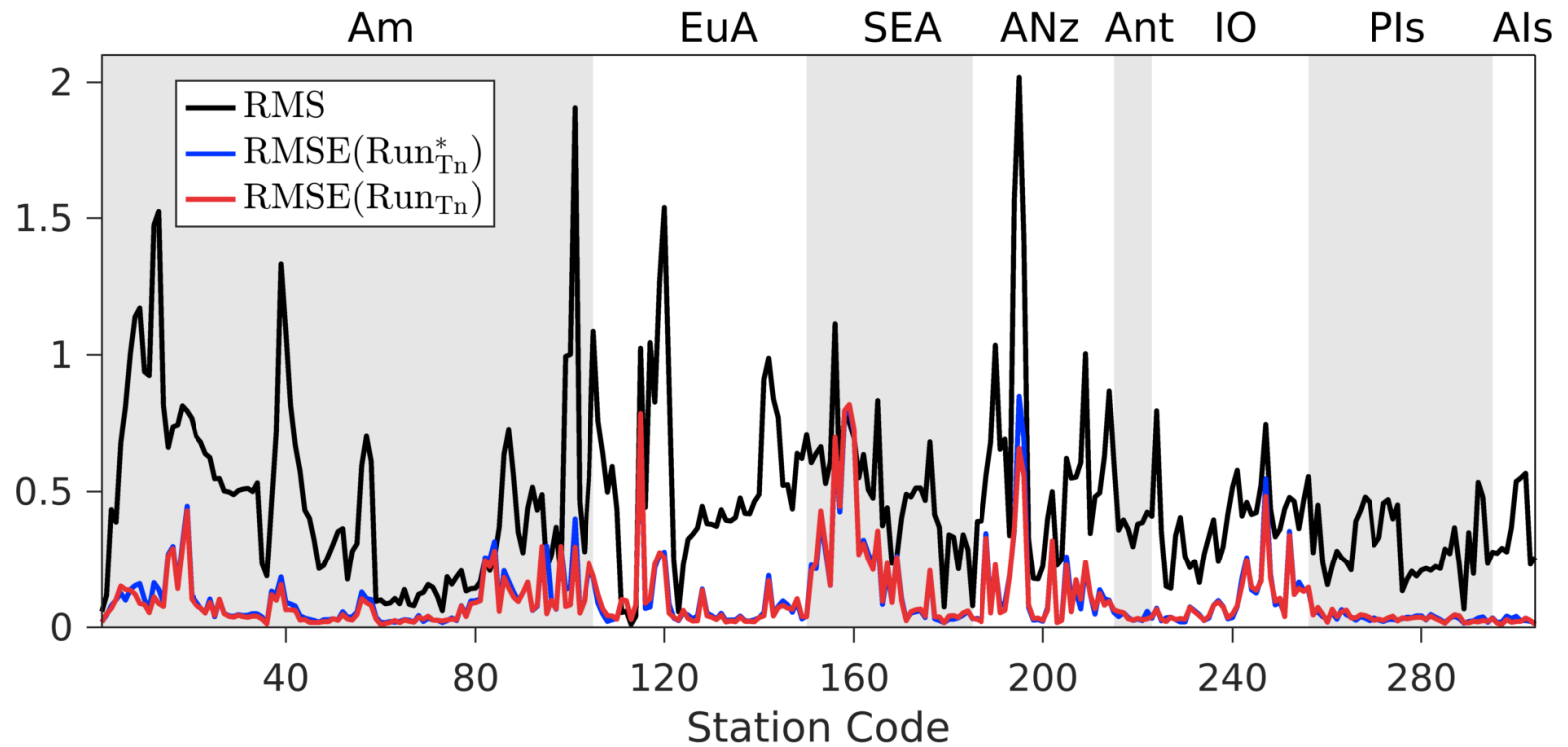
$$|Z_{obs} - Z_{mod}|$$

$$\tilde{\gamma}^2 = \frac{\int_0^p |\tilde{\mathbf{u}}_{obs}(t) - \tilde{\mathbf{u}}_{mod}(t)|^2 dt}{\int_0^p |\tilde{\mathbf{u}}_{obs}(t)|^2 dt}$$



As nudging η violates mass conservation which may create inconsistency between η and u , a fair comparison is to compare  simulated tidal currents (bottom panel)

Nudging u VS. Nudging η (Comparison with tide gauge data)



Overall, the comparison of tidal currents from TPXO8 and tides at gauges demonstrate that nudging u is the best approach.



Predicting the tides

- RMS_{50} : median of RMS values for observed tides at 304 gauges.
- RMSE_{50} : median of RMSE values for runs without and with tidal nudging

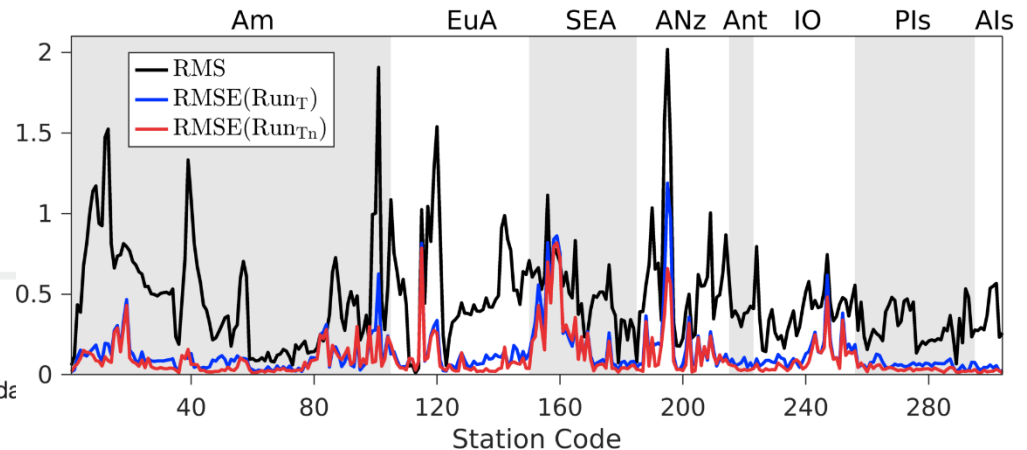
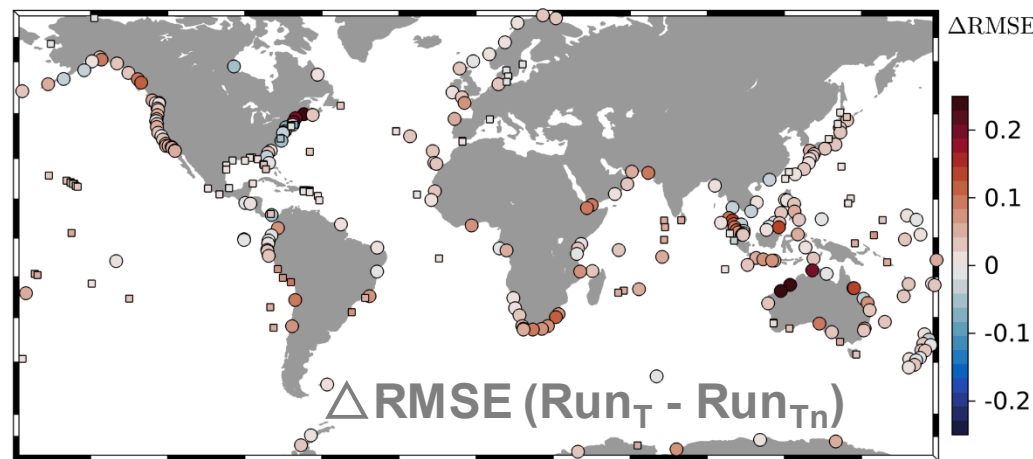
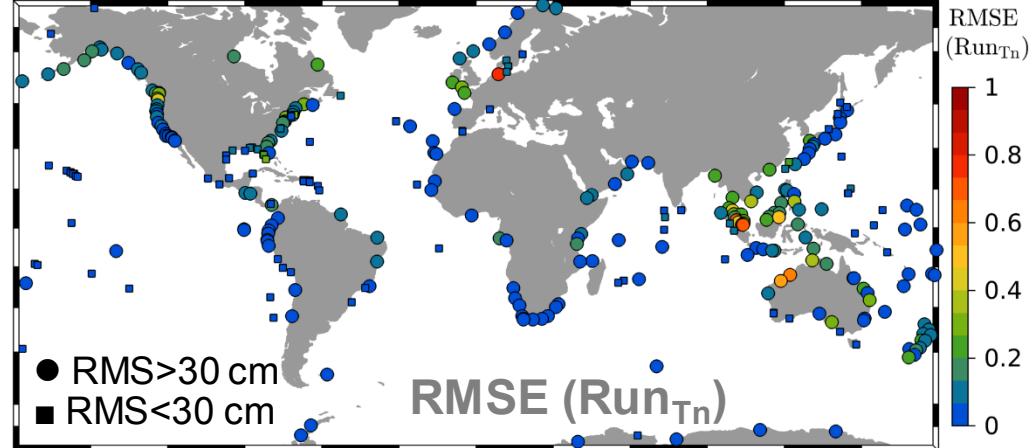
| | O_1 | K_1 | P_1 | Q_1 | M_2 | S_2 | N_2 | K_2 | S_2 | All |
|-------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| RMS_{50} | | 0.143 | | | | 0.365 | | | 0.118 | 0.416 |
| $\text{RMSE}_{50} \text{ Run}_T$ | | 0.028 | | | | 0.077 | | | 0.035 | 0.086 |
| $\text{RMSE}_{50} \text{ Run}_{Tn}$ | | 0.023 | | | | 0.040 | | | 0.013 | 0.053 |

The impact of nudging is most drastic for S_2 . One reason is that Run_{Tn} includes rS_2 through the nudging to TPXO8, consistent with tide gauge data which also include this rS_2 signal.

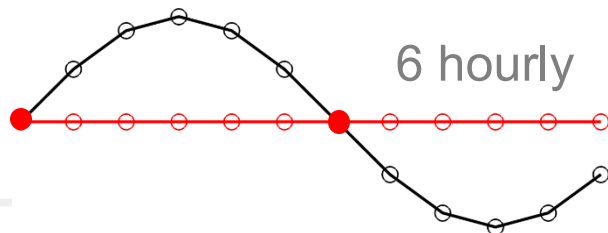
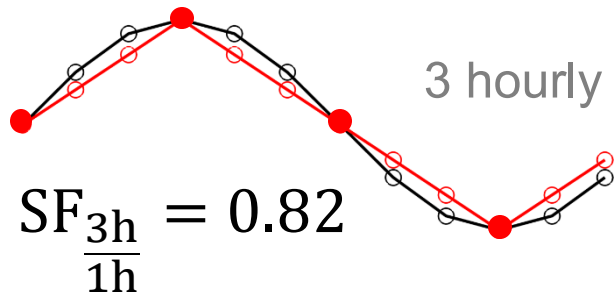
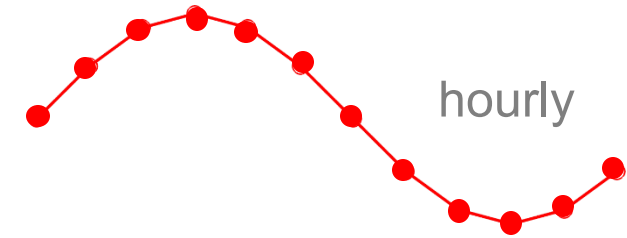


Predicting the tides

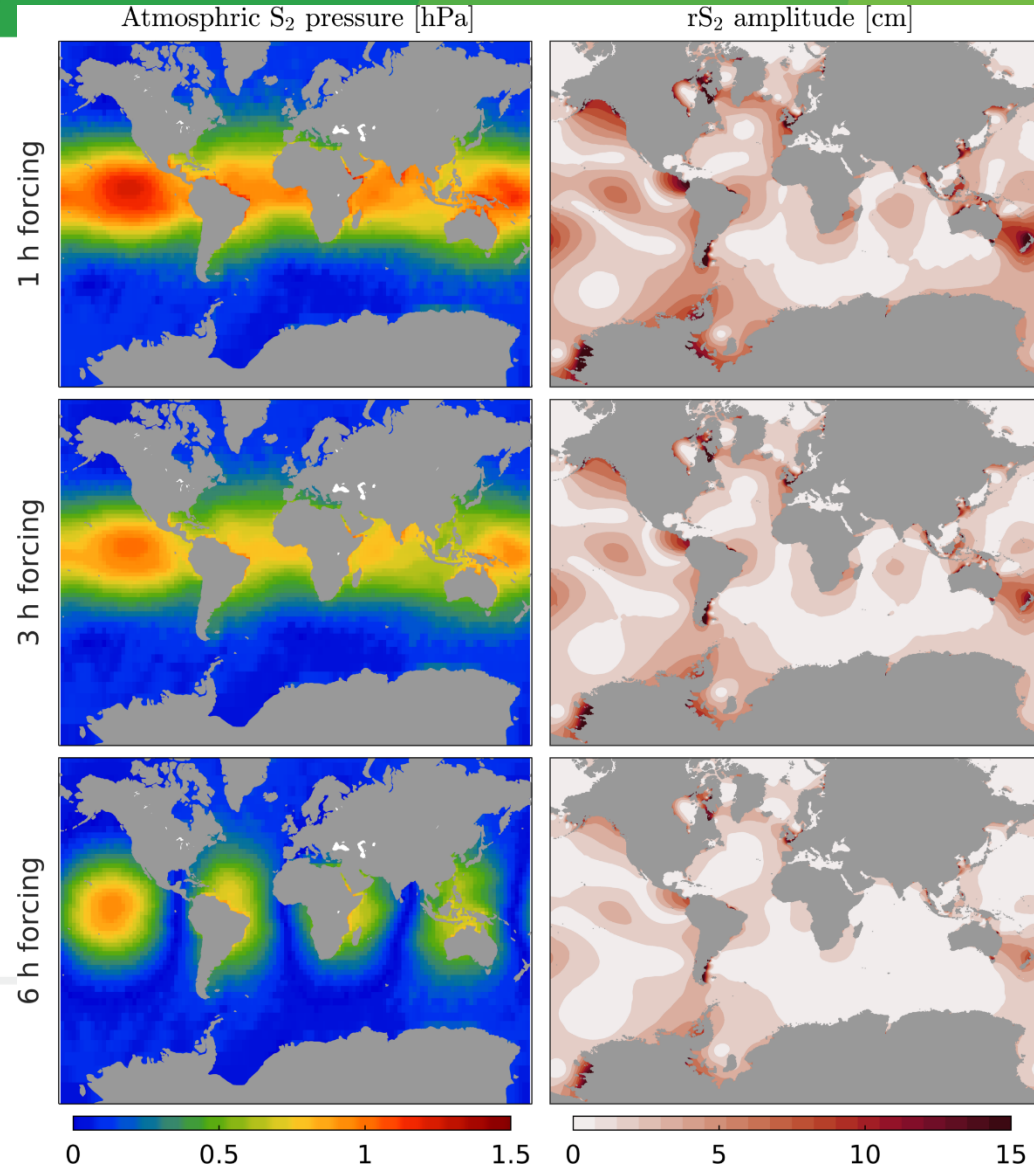
- Tidal nudging improves the model skill at 82% of the 304 stations, and reduces the average RMSE by 23% (from 0.13 m to 0.10 m).
- Comparable to data-assimilative model FES2012 in terms of average RMSE (Muis et al., 2016)



Sea level response to S_2 air pressure and need for hourly forcing

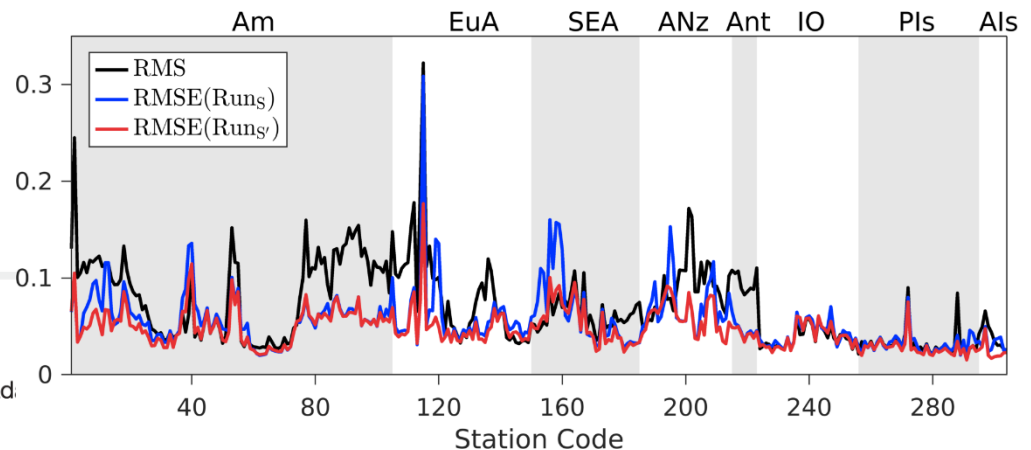
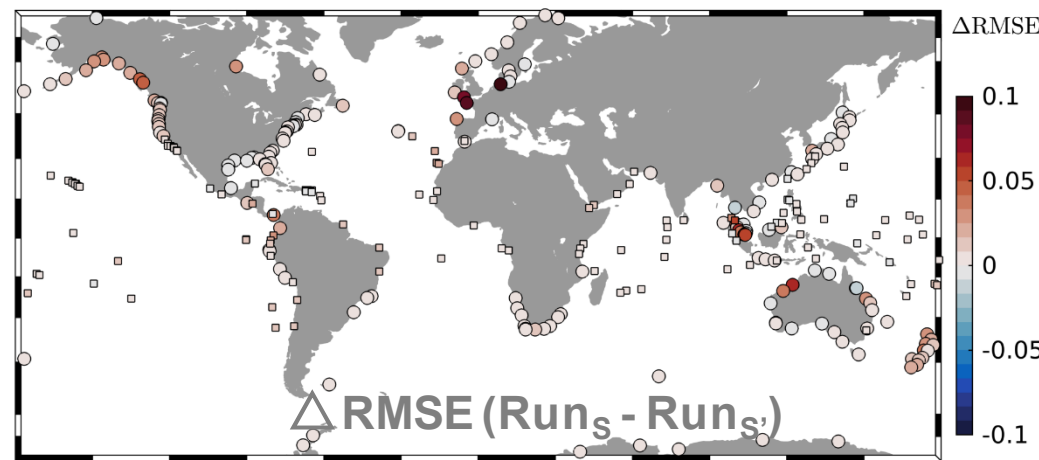
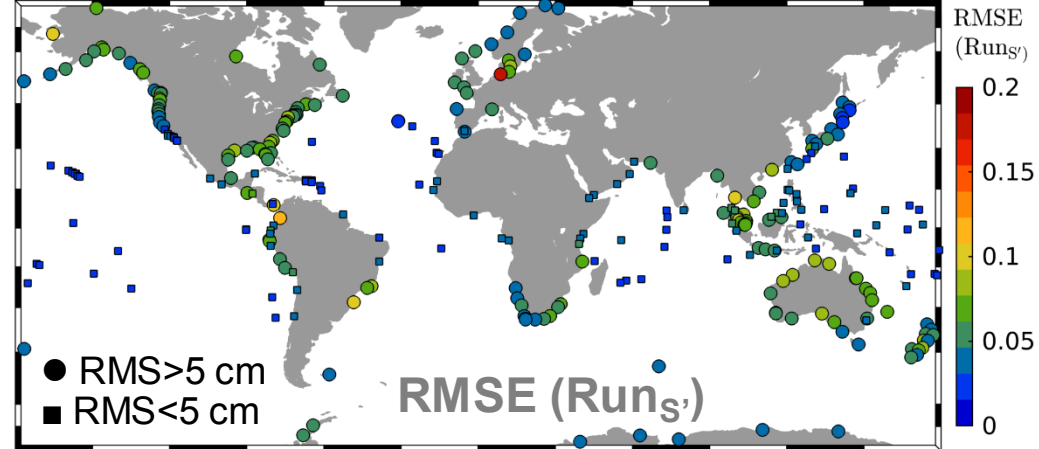


$SF_{\frac{6h}{1h}} = 0.82|\sin(\varphi)|$ Canada

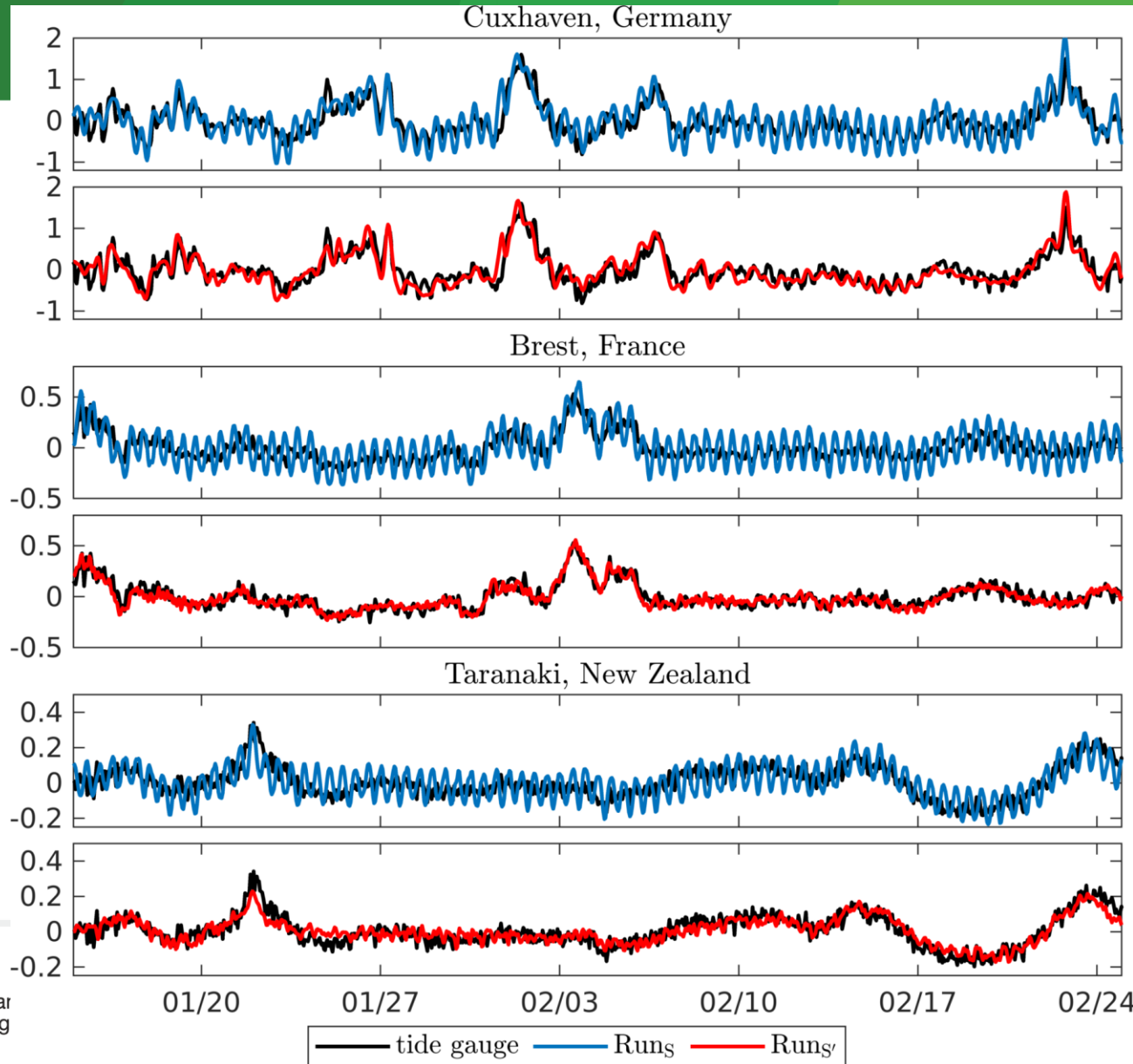


Predicting the surges

- rS_2 in Run_S needs to be removed to be consistent with tidal residuals in which rS_2 is also removed by t_{tide} .
- rS_2 is removed by removing the S_2 component from the forcing, which is $Run_{S'}$
- Low frequency (>20 days) signals are filtered out

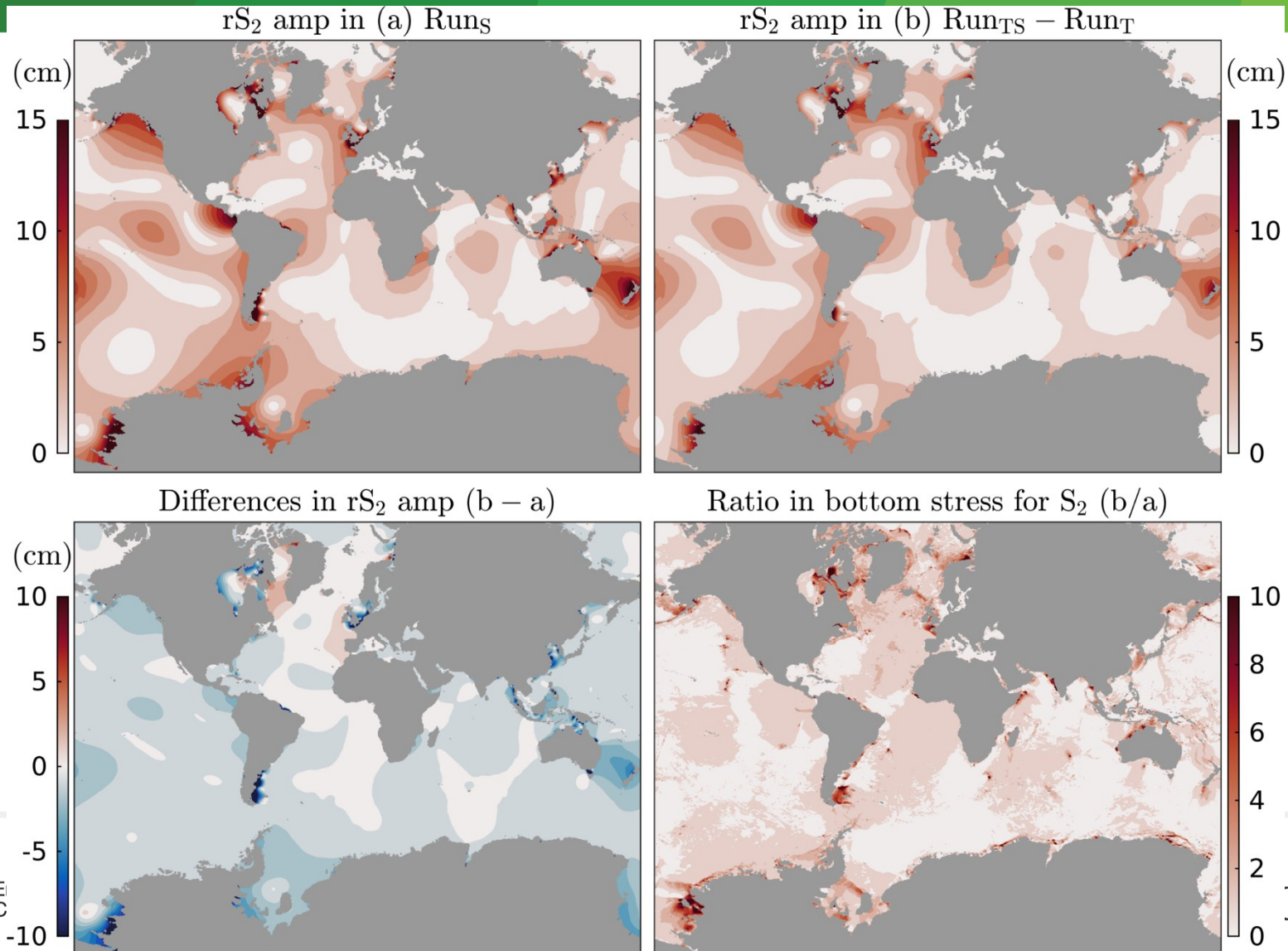


Time series of observed and predicted surge level at three selected stations



Damping of rS_2 by the gravitational tide

If a current is a combination of tidal components, then bottom friction at a given tidal frequency can be increased by other tidal components



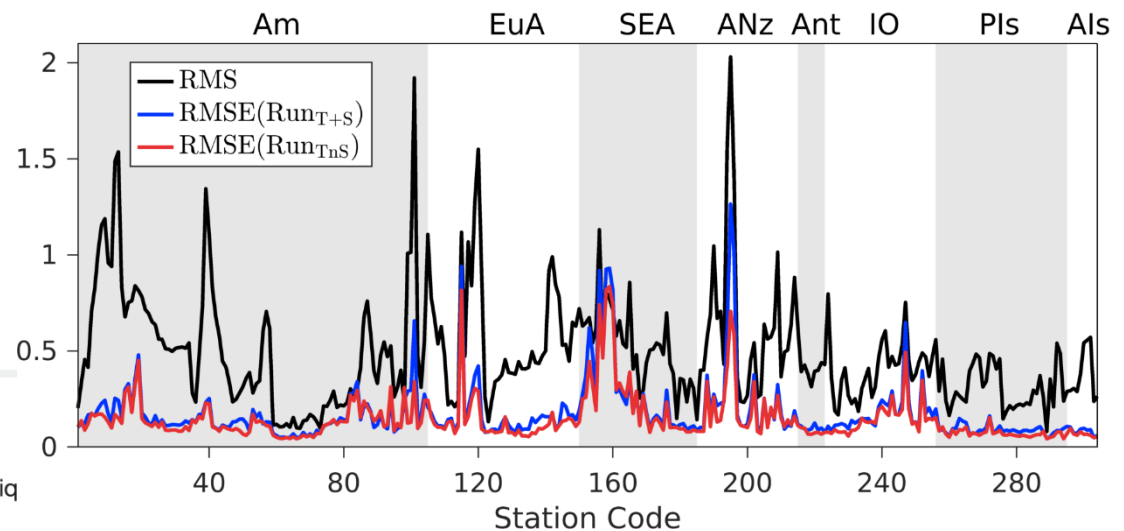
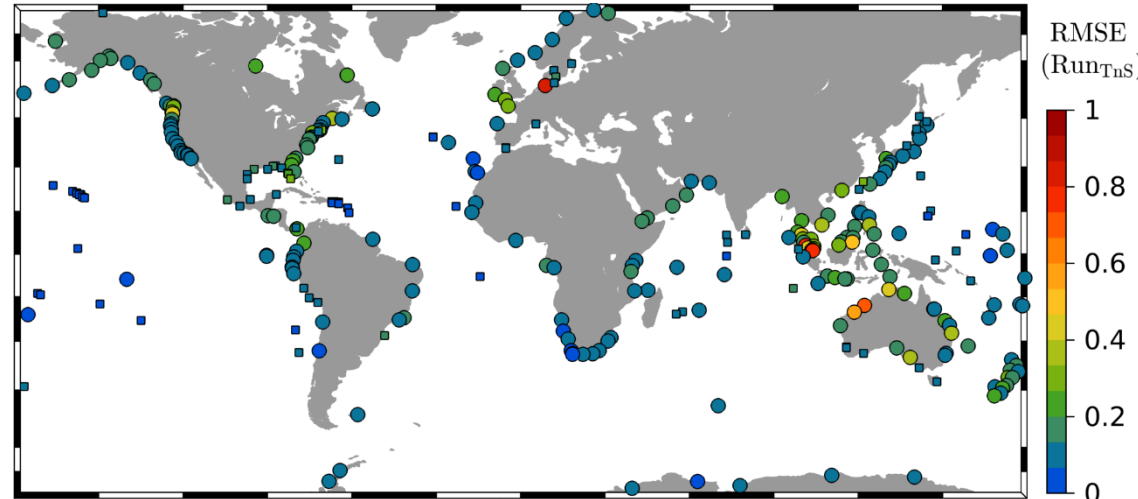
Predicting the total water level

| | |
|---------------------------|------------------------------------------------------|
| Run_{T+S} | No tidal nudging; No nonlinear interaction |
| Run_{TS} | No tidal nudging |
| Run_{Tn+S} | No nonlinear interaction; Double counting of rS_2 |
| Run_{TnS} | Include both tidal nudging and nonlinear interaction |

| | Unfiltered | Subseasonal $480 > p$ | Diurnal $30 > p > 18$ | Semi-diurnal $16 > p > 9.6$ | |
|---------------------------|------------|--------------------------|--------------------------|--------------------------------|------------------|
| Obs | 43.4 | 42.6 | 13.8 | 36.4 | } Median of RMS |
| Obs - Run _{T+S} | 12.5 | 10.3 | 3.3 | 8.5 | |
| Obs - Run _{TS} | 12.3 | 10.3 | 3.2 | 8.4 | } Median of RMSE |
| Obs - Run _{Tn+S} | 11.2 | 8.2 | 2.9 | 5.7 | |
| Obs - Run _{TnS} | 10.7 | 8.0 | 2.9 | 5.1 | |

Predicting the total water level

- No filter is applied, only the mean is removed.
- RMSE below 0.20 m for 83% of the stations
- The average RMSE in Run_{TnS} is 0.15 m. For comparison, it is 0.17 m in Muis et al., (2016). Note that tide gauges and analysis periods in the two studies are different.



CONCLUSIONS

- **Tidal nudging** in deep water only is shown to improve tide prediction at the coast.
- **Hourly atmospheric forcing** is required to resolve the radiational S_2 tide (rS_2).
- rS_2 is subject to strong **nonlinear interaction** with gravitational tides.
- Due to this nonlinear interaction, it is necessary to use the **coupled tide-surge run** for global operational forecasting and climate sensitivity studies.

