An adaptive coordinate for ocean modelling

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With thanks to: Alistair Adcroft, Bob Hallberg

Adaptive/hybrid coordinates

- **Hybrid: z-tilde (Leclair & Madec)**
  - Lagrangian on short timescales, but long timescale relaxation
- **Adaptive: HYCOM (Bleck)**
  - Isopycnal with minimum layer thickness
- **Adaptive: Burchard & Beckers**
  - A combination of different components (stratification, shear, near-surface, background)

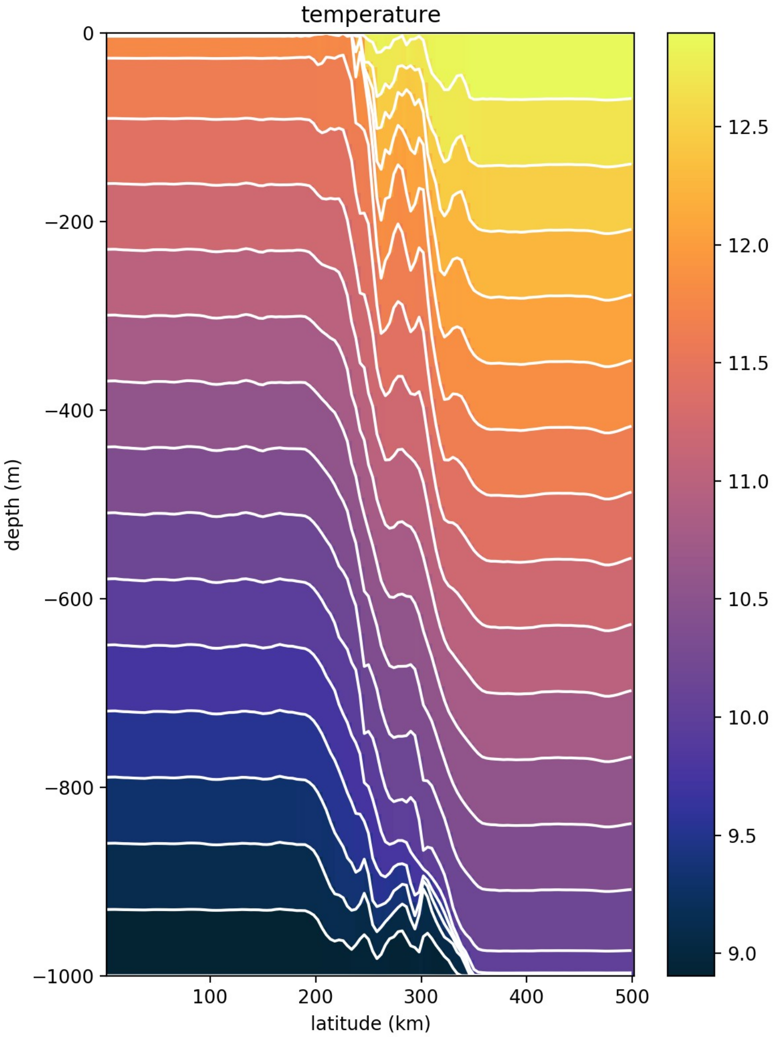
A quick primer/refresher on ALE

Designing an adaptive vertical coordinate

- **Horizontal discretisation is fixed**
  - Almost always a regular/structured grid
- **Full control of vertical coordinate**
  - We can change it while the model is running
  - Take advantage of this!
- **Adaptivity: dynamically adjusting the coordinate according to the actual state**

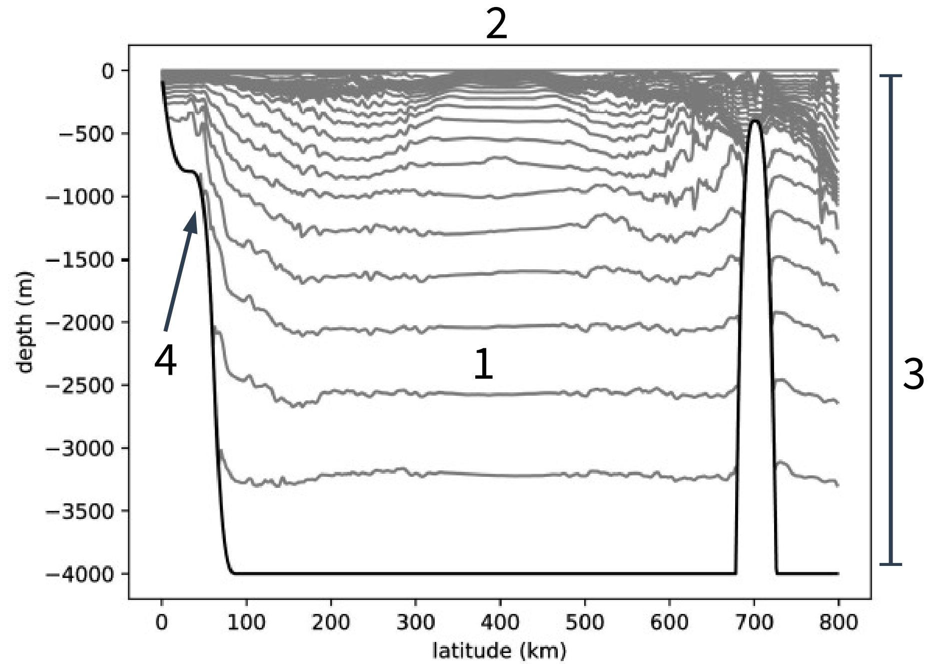
Core principles

- **Give similar interior to isopycnal coordinate**
  - But still flexible to surface mixing, convection, etc.
- **Make better use of layers**
  - We only need to be *locally* isopycnal



The recipe

1. **Density adaptivity**
   - Locally isopycnal interior
2. **Lateral smoothing**
   - Weakly stratified regions
3. **Vertical restoring**
   - Overall structure
4. **Grid adjustment**
   - Prevent numerical instabilities

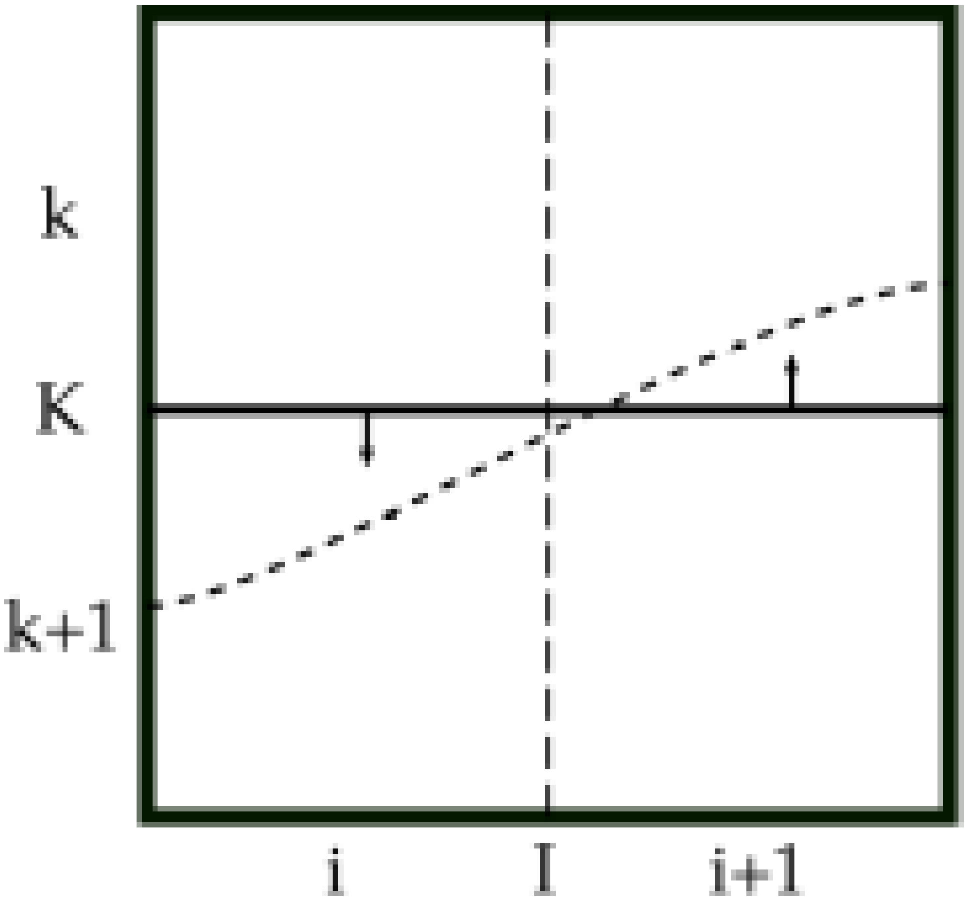


$$\partial_t z_k = -\nabla_H \cdot \left(\underbrace{\omega_\sigma \frac{\kappa \nabla_H \sigma}{\sqrt{\sigma_z^2 + (\nabla_H \sigma)^2}}}_{\text{density adaptivity}} + \underbrace{\omega_z \kappa \nabla_H z_k}_{\text{lateral smoothing}} \right) + \underbrace{\tau_r^{-1} \left(z_k^* - z_k \right)}_{\text{vertical restoring}} + \underbrace{F_{\text{con}}}_{\text{grid adjustment}}$$

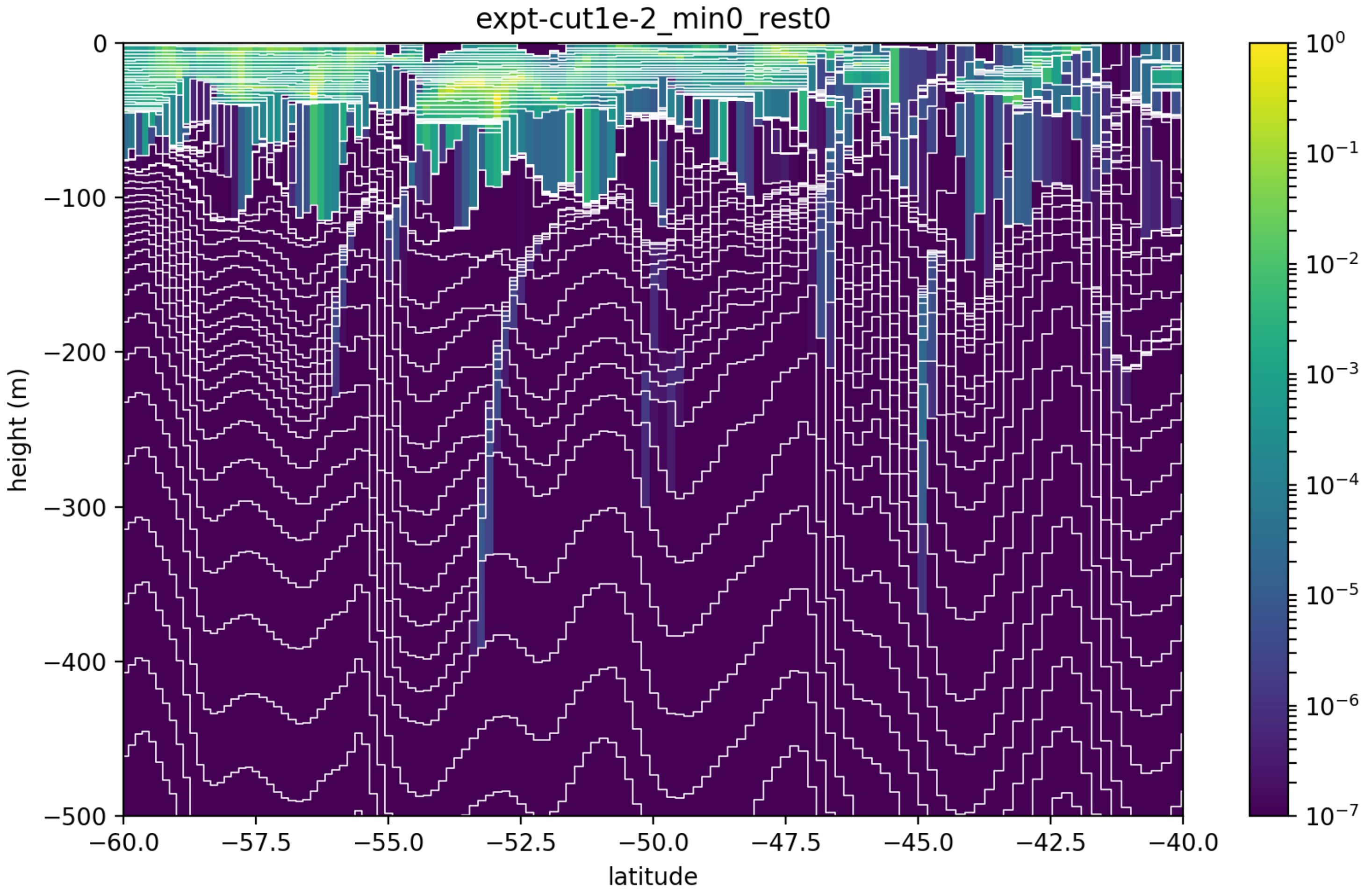
1. Density adaptivity

- **Adjust interface (solid line) to an isopycnal (dashed line)**
- **Flux form: conserves mean layer height**
  - Don't end up solving for the wrong isopycnal!

$$F_{\sigma_x} = \omega_\sigma \kappa \frac{\partial_x \sigma}{\sqrt{(\partial_z \sigma)^2 + |\nabla_H \sigma|^2}}$$



2. Lateral smoothing



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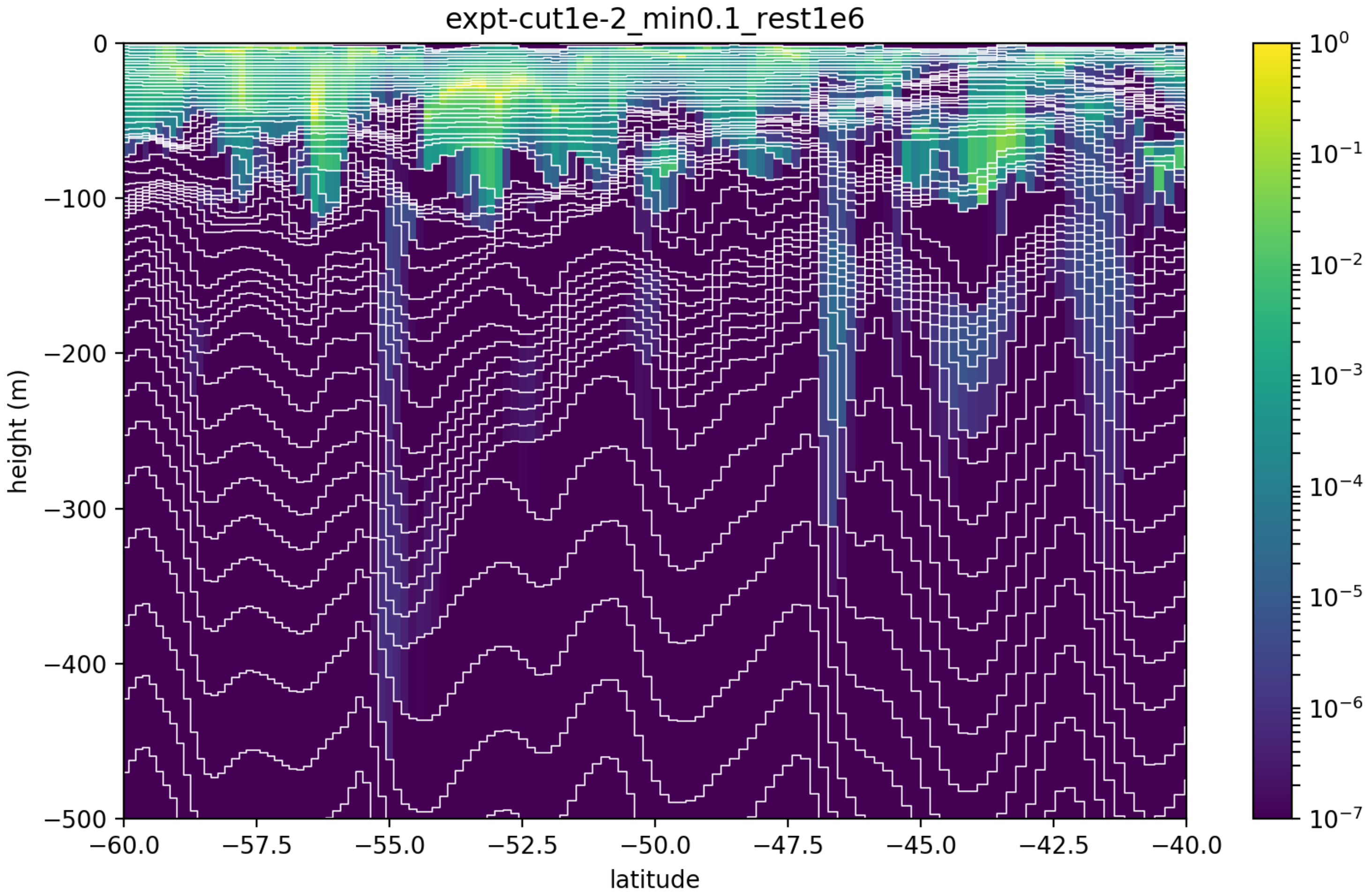


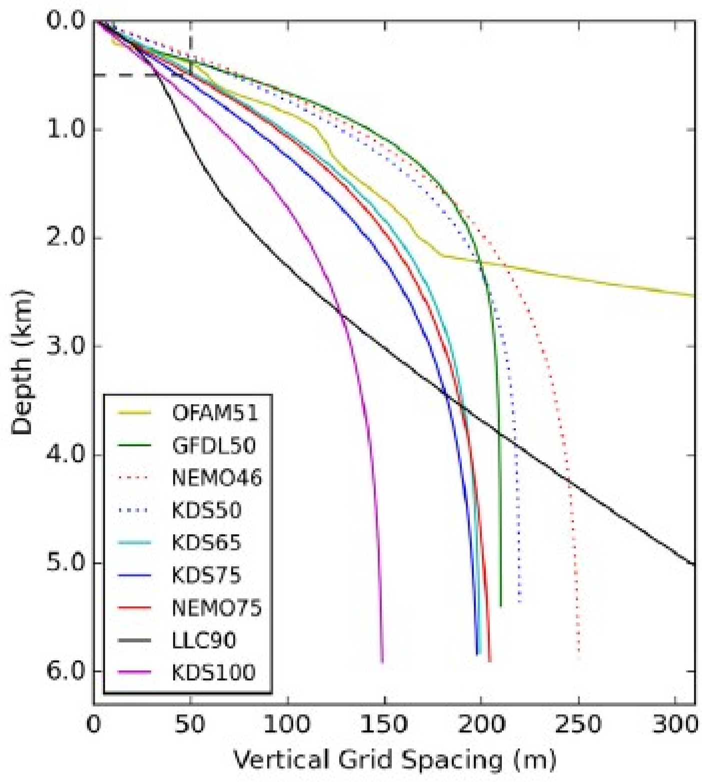
2. Lateral smoothing

- **Where stratification is weak, isopycnals aren't well-defined**
  - Smooth out the coordinate in these regions (particularly surface)
- **Important behaviour: breaks "tangles"**
  - We always use a little bit of smoothing

$$F_{z_x} = \omega_z(\Delta x^2/\Delta t)\delta_i z_k$$

3. Vertical restoring

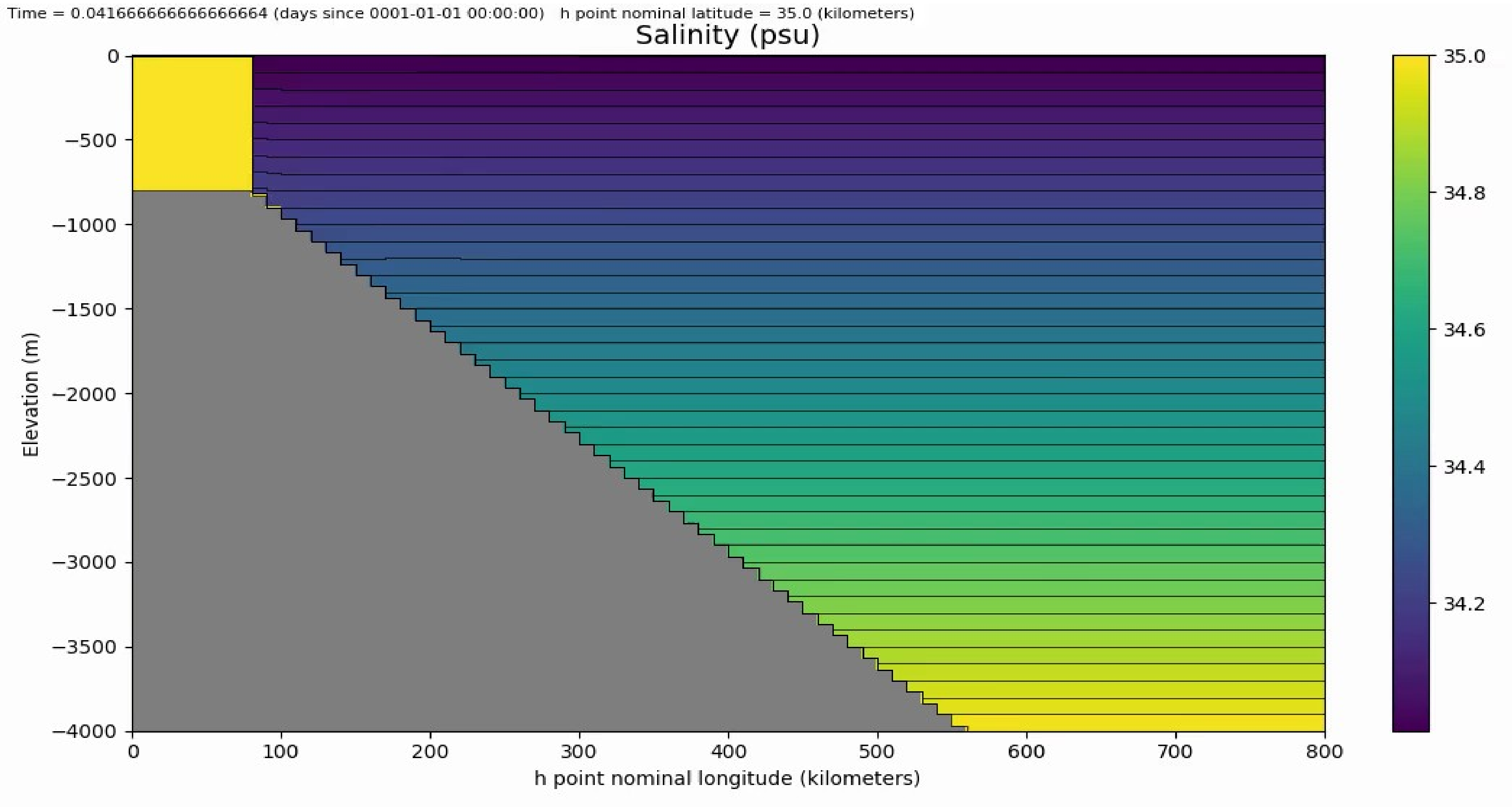
- Density adaptivity operates at small scales
- Restoring gives overall structure through the full depth – importantly the surface
- Gently restore towards a target coordinate (long timescale)



Stewart et al, 2017

$$F_r = \tau_r^{-1}(z_k^* - z_k)$$

4. Grid adjustment



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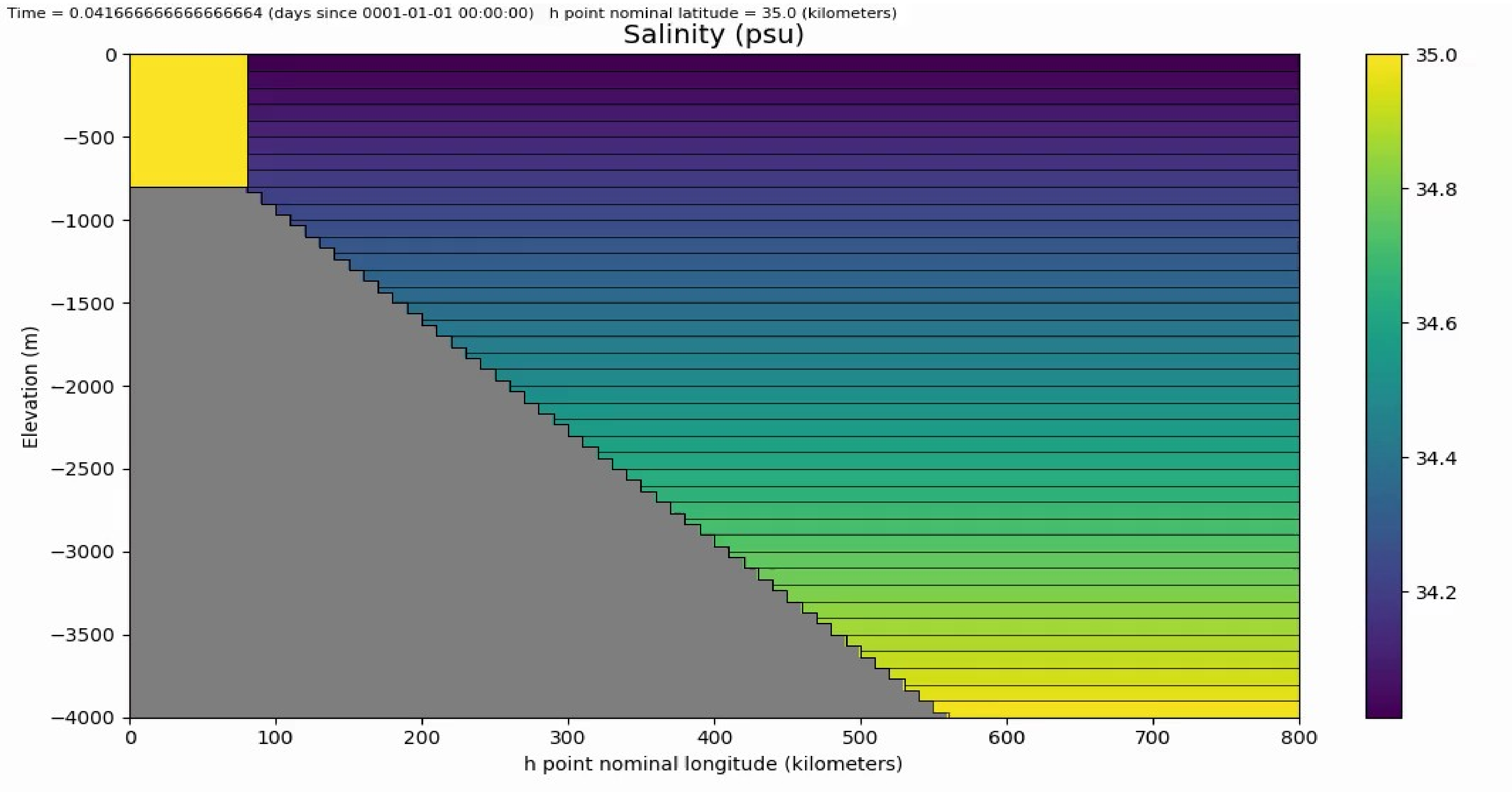


4. Grid adjustment

- **Preemptively change the coordinate when:**
  - We know the solution will be unphysical
  - We are free to do so (moving mass in/out of vanished layers doesn't mix)
- **Make use of the direction of advection**
- **Ensure that thin layers don't have a disproportionate effect!**

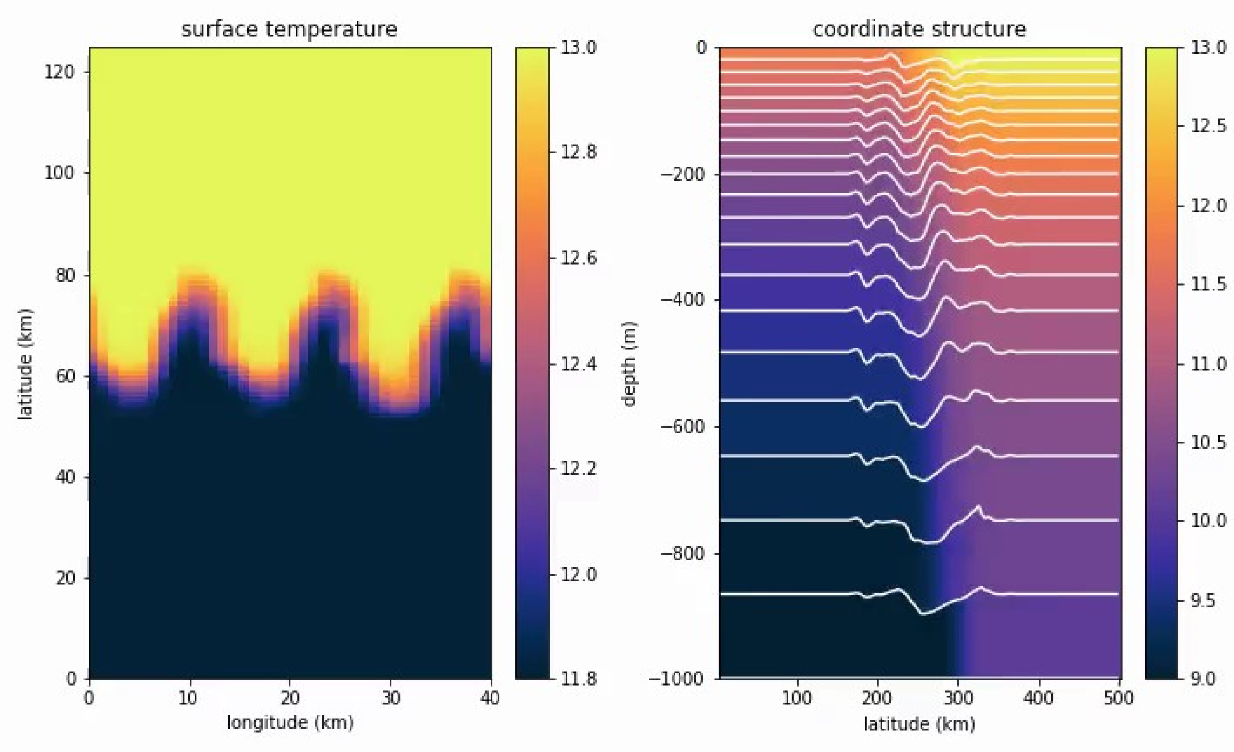
$$\Delta z = \alpha \frac{u \Delta t}{\Delta x} (k_t - k) L$$

4. Grid adjustment

- **Algorithm:**
  - Look at the column downstream
  - If advection would be unstable, move the bottom interface up (or the top down)
  - Subject to a CFL constraint (we only want to move through thin/vanished layers)
- **We try to prevent the mixing of unrelated water masses**

Example

- Unstable, eddying channel
- Coordinate follows intrusions from eddies
- Evaluation of the coordinate is forthcoming



Conclusions

- **Developed potential energy diagnostic for horizontal/vertical spurious mixing**
  - Motivated improvement to vertical coordinate
- **Developed an adaptive vertical coordinate**
  - Dynamic, responds to what is actually happening
  - Density adaptivity (isopycnal-like)
  - Balanced out by smoothing/restoring
  - Further non-destructive adjustment

Questions?