# A Primer on the Vertical Lagrangian-Remap Method in Ocean Models Based on Finite Volume Generalized Vertical Coordinates

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# My biased points of view

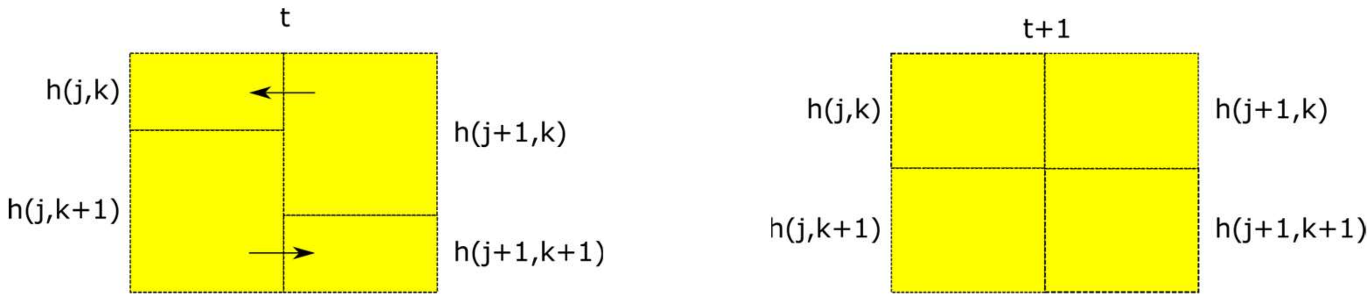
- More experience with isopycnal models (GOLD) and MOM6
- NEMO (within CanESM5) is my first experience with z-like GCM and ALE implementation
- My 'default' view of ocean models
  - Depth is a purely diagnostic quantity
  - Continuity equation is primarily responsible for determining thickness fluxes
  - Strong dynamical split between separation between barotropic and baroclinic dynamics
  - Timestep consists of intermediate states (no accumulation of tendencies)

# Brief summary of paper

- Section 2 (not discussed)
  - Introduces notation
  - Discusses the weak (integral) formulation of the primitive equations in general coordinates
- Section 3 (not discussed)
  - Construction of cell budgets for momentum and scalar quantities in Finite-Volume
- Section 4
  - Description of what the Lagrangrian Regrid/Remap step entails
- Section 5
  - Comparison of thickness and tracer equations in quasi-Eulerian formulation, NEMO-like ALE (Madec 2008), and the Vertical Lagrangian Remap (after Hirt et al. 1974)

# Similarities of ‘Lagrangian Vertical Remapping’ Method to isopycnal models

- Some analogues to layered isopycnal models
  - Grid evolves in a ‘Lagrangian’ way due to changes in layer thickness
- Isopycnal coordinate, the ‘vertical velocity’ is constrained by transport of heat/salt to preserve potential density
  - No such constraint for arbitrary coordinate



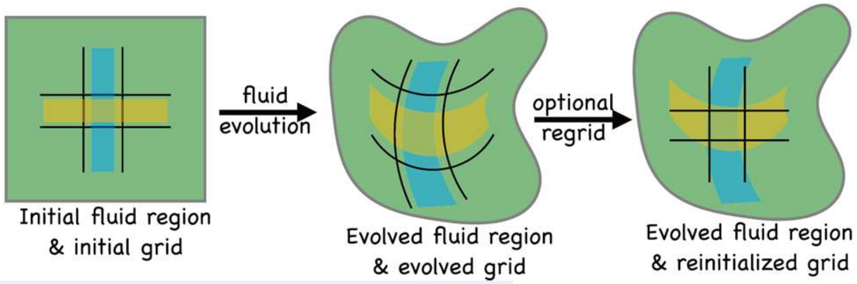

- In the Lagrangian limit, all grid motions follow fluid motions
  - **No motion of fluid relative to grid**

# Vertical motions of grid vs. fluid

- Two vertical velocities to consider
  - Vertical motion of the grid $w^{\text{grid}}$
  - Vertical motion of the fluid across a moving model surface $w^{(\dot{s})}$

NEMO-style ALE

| | |
|---|---|
| $\Delta_s w^{\text{grid}} = (h^{\text{target}} - h^{(n)})/\Delta t$ | General layer motion |
| $\Delta_s w^{(\dot{s})} = -\Delta_s w^{\text{grid}} - \nabla_s \cdot [h\boldsymbol{u}]^{(n)}$ | Diagnose dia-surface transport |
| $h^{\dagger} = h^{(n)} - \Delta t \nabla_s \cdot [h\boldsymbol{u}]^{(n)}$ | Horz advection thickness update |
| $h^{(n+1)} = h^{\dagger} - \Delta t \Delta_s w^{(\dot{s})}$ | Vert advection thickness update |




Vertical Lagrangian remap: Coordinate free

| | |
|---|---|
| $\Delta_s w^{\text{grid}} = -\nabla_s \cdot [h\boldsymbol{u}]^{(n)}$ | Layer motion via convergence of horz advection |
| $h^{\dagger} = h^{(n)} + \Delta t \Delta_s w^{\text{grid}} = h^{(n)} - \Delta t \nabla_s \cdot [h\boldsymbol{u}]^{(n)}$ | Horz advection thickness update |

- For $\tilde{z}$: target grid chosen to ‘absorb’ high-frequency oscillations
- **These two equations are identical in the Lagrangian limit: $w^{\dot{s}} \rightarrow 0$**

# Regridding and remapping in place of vertical tracer advection

| | |
|---|---|
| $h^{(n+1)} = h^{\text{target}}$ | Regrid to the target grid |
| $\Delta_s w^{(\dot{s})} = -(h^{\text{target}} - h^{\dagger})/\Delta t$ | Diagnose dia-surface transport |
| $[h\,C]^{(n+1)} = [h\,C]^{\dagger} - \Delta t\,\Delta_s(w^{(\dot{s})}\,C^{\dagger})$ | Remap tracer using dia-surface transport |

- **Regridding** refers to the construction of a new grid based on prognostic changes in the water column
  - In FV models: Create a new set of layer thicknesses based on some grid definition
  - These grids *do not need* a physical interpretation, but it can be ***useful***
  - Example:
    - Based on the Lagrangian evolution of temperature, salinity, and thickness construct a a new column whose interfaces are surfaces of potential density
- **Remapping** refers to the transformation of the model state from one grid to the other
  - Equations above are misleading, the diagnosis of dia-surface transport is *not needed*
  - The effective dia-surface transport has to be backed out of remapping

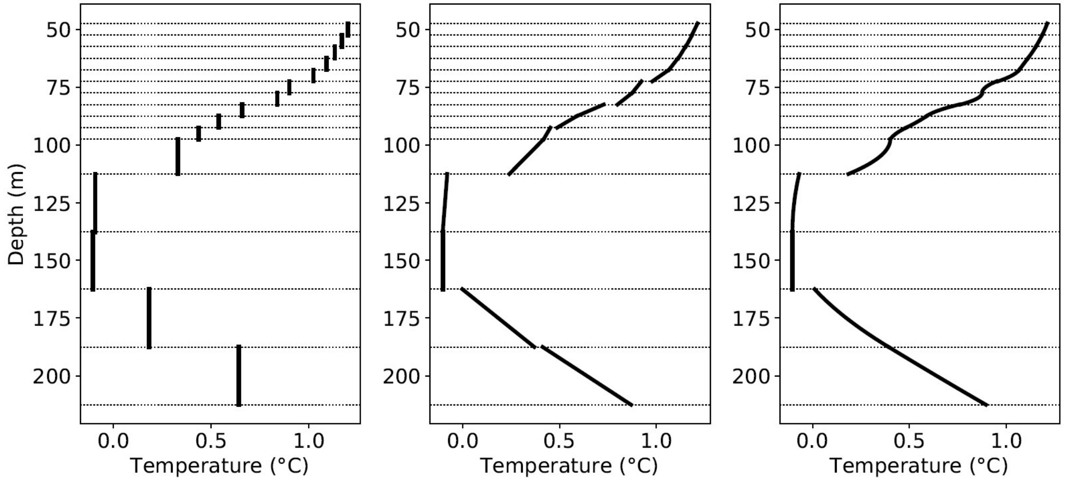
# Features of regridding/remapping

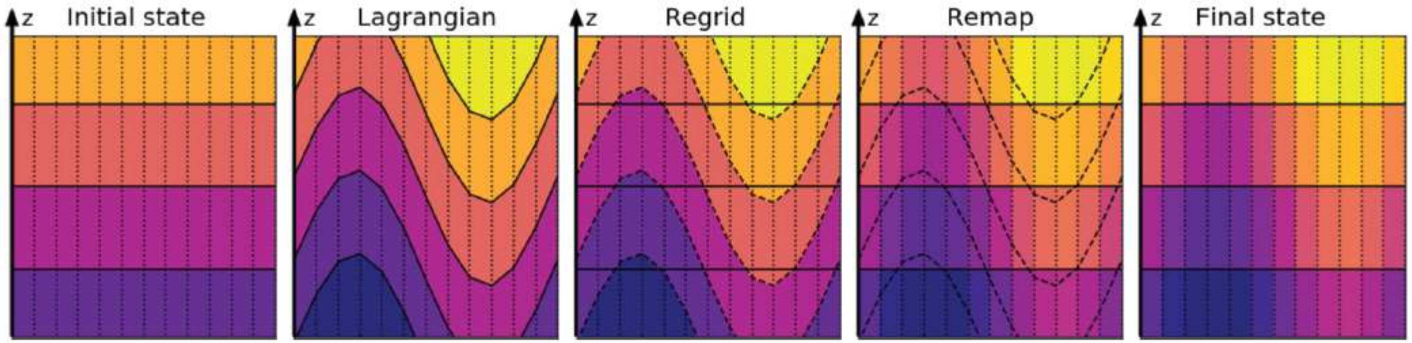
- Can be cast as a type of *interpolation*
  - No vertical limit on CFL
  - Wetting and drying can be handled by inflating/deflating ‘vanished’ layers
- Vertical ‘transport’ of tracer only occurs during remapping
- Numerical truncation error depends primarily on the accuracy of ‘reconstructions’ (up to $5^{th}$-order accurate schemes are available)
- Vertical coordinate can be defined by a grid generator
  - State-dependent vertical coordinates
    - Isopycnal, hybrid, density-slope minimizing (e.g. Gibson Thesis)
- The number of grid cells in the vertical can vary
  - Allowing for adaptive algorithms?
- **Caveat:** The grid constantly evolves after the regrid/remap, e.g. no guarantee that model surfaces remain isopycnal-like or z*

# Demonstration of regrid/remapping approach

1. Model layers evolve due to dynamics
2. Generate a target grid
3. Create polynomial reconstructions for velocities and tracers (e.g. White and Adcroft 2008 and Engwirda)
4. Use reconstructions to remap from old grid to new grid

Piecewise Constant | Piecewise linear | Piecewise parabolic



# VLR has three distinct groups of processes

Vertical Lagrangian remap: Coordinate free

| | | |
| --- | --- | --- |
| $\Delta_s w^{\text{grid}} = -\nabla_s \cdot [h\boldsymbol{u}]^{(n)}$ | Layer motion via convergence of horz advection | Dynamics (fastest timescales) |
| $h^{\dagger} = h^{(n)} + \Delta t \, \Delta_s w^{\text{grid}} = h^{(n)} - \Delta t \, \nabla_s \cdot [h\boldsymbol{u}]^{(n)}$ | Horz advection thickness update | Dynamics (fastest timescales) |
| $[hC]^{\dagger} = [hC]^{(n)} - \Delta t \, \nabla_s \cdot [hC\boldsymbol{u}]^{(n)}$ | Horz advection tracer update | Horizontal tracer advection |
| $h^{(n+1)} = h^{\text{target}}$ | Regrid to the target grid | Vertical tracer transport |
| $\Delta_s w^{(\dot{s})} = -(h^{\text{target}} - h^{\dagger})/\Delta t$ | Diagnose dia-surface transport | Vertical tracer transport |
| $[hC]^{(n+1)} = [hC]^{\dagger} - \Delta t \, \Delta_s (w^{(\dot{s})} C^{\dagger})$ | Remap tracer using dia-surface transport | Vertical tracer transport |

- In between Dynamics and Advection, mass transports are accumulated
- In between Regrid/Remap steps, the layer thicknesses continue to evolve in a Lagrangian way
- These can be subcycled with different timesteps and still maintain consistency

# Topics for discussion on implementing VLR in NEMO

- Fundamental equations for both VLR and ALE converge in the Lagrangian limit
  - VLR-mode is a 'special case' and not a complete overhaul
- Intermediate steps:
  - (Bad?) idea: split tracer vertical advection from thickness/velocity advection?
  - Another (better?) idea: Focus on prototyping routines for diagnostic transformations
    - In MOM6, core ALE algorithms are used to remap from HYCOM to isopycnal, z*. All tracer content budget terms close
- Is the implicit nature of the regrid/remap a downside?
  - Vertical diffusion already is its own implicit update
  - What about back-calculating tendencies? $\frac{c_{new}h_{new}-c_{old}h_{old}}{\Delta t}$
- Can we incorporate ideas about $\tilde{z}$ into a VLR framework?
  - Target grid for remapping can be 'filtered' in time