Accurate calculation of pressure forces using steeply sloping coordinates

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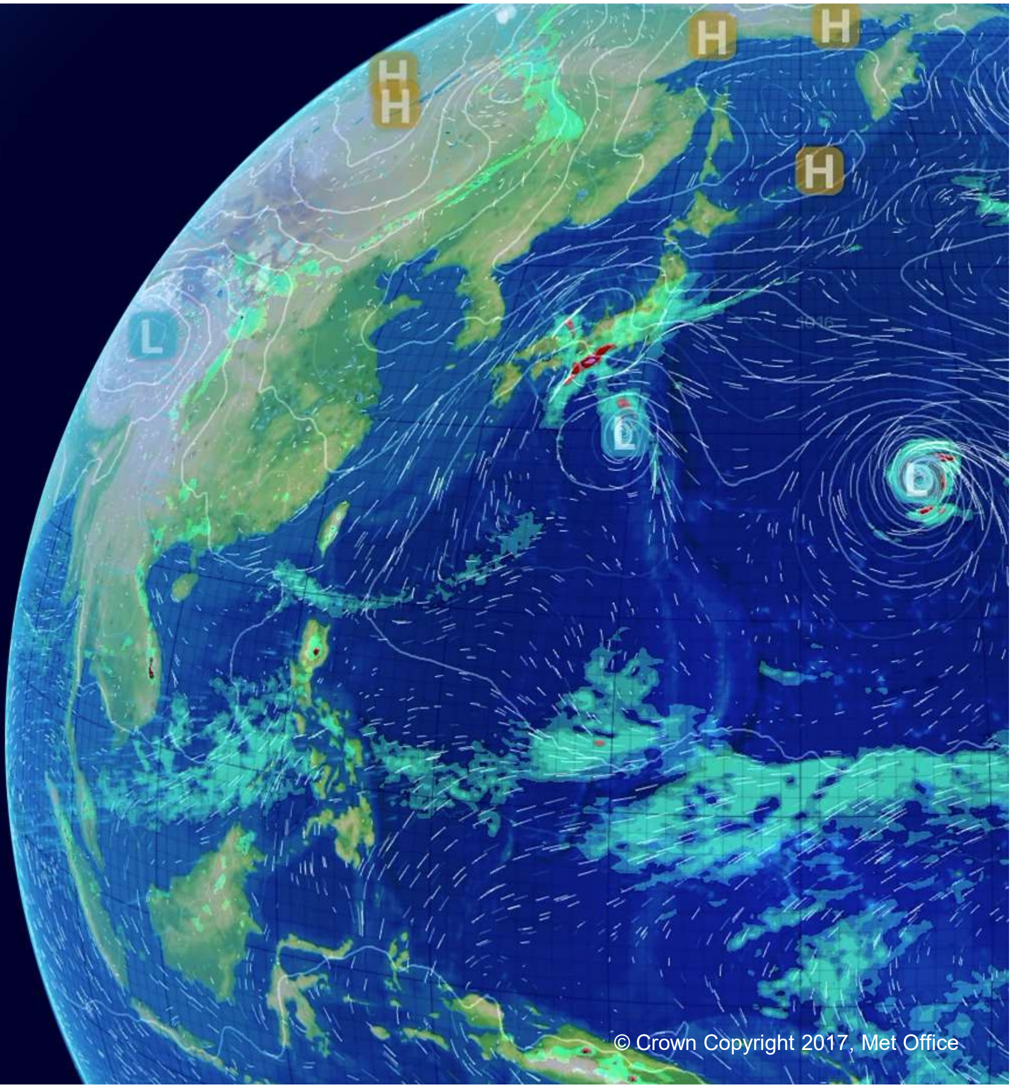

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Content

1. Motivations
2. Introduction to the main ideas
3. Some initial results (higher order accuracy is needed)
4. Open / unresolved issues
5. Summary

Note: “Sec #” refers to section # of Bell et al. (27 July 2018 version)

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1. Motivations (sec 1)

Stepped bathymetry has:

- a poor representation of overflows
- uneven (hence noisy) vertical velocities
- unclear implications for vortex stretching
- step-like side-walls along the continental slope

Is it possible to represent pressure forces accurately enough using terrain-following coordinates over steep slopes ?

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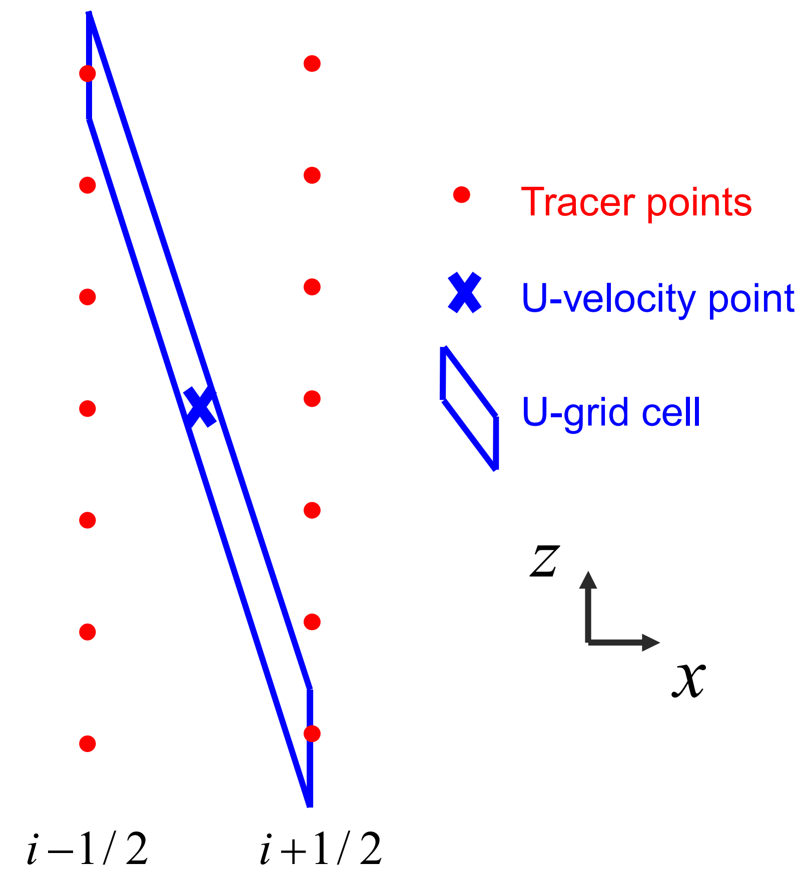
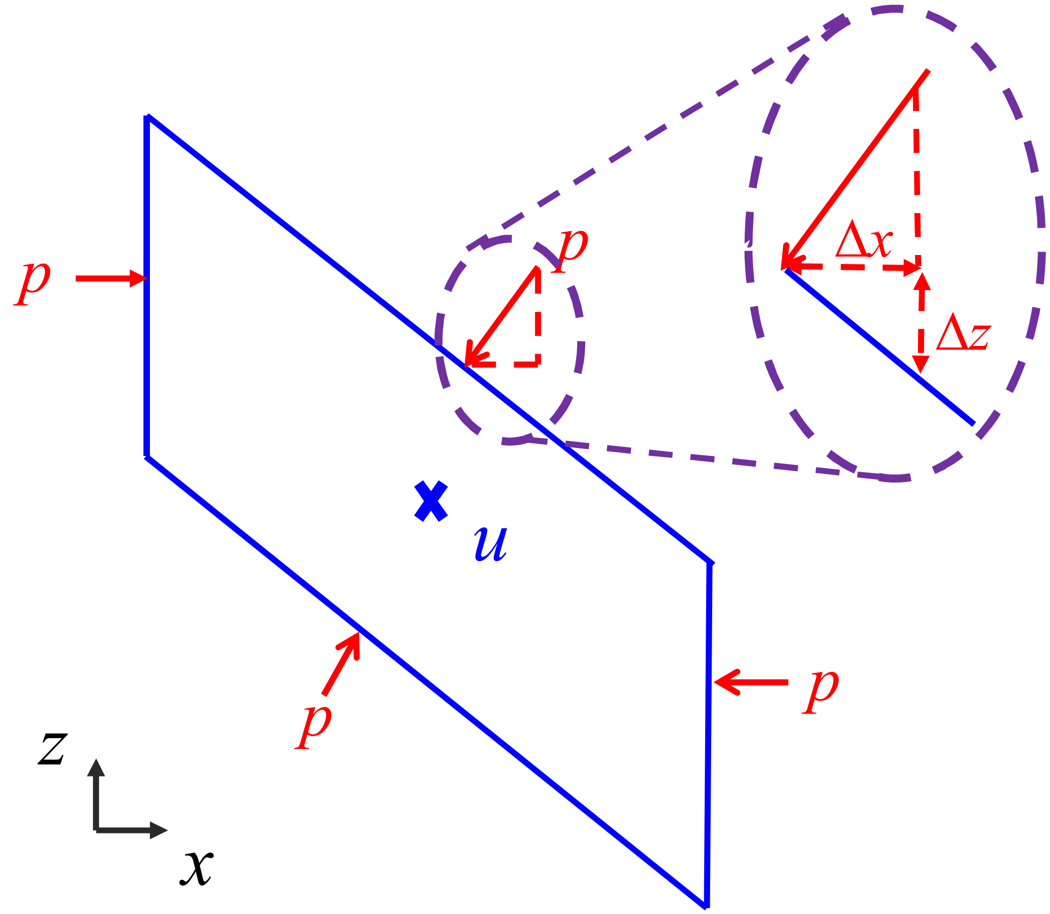
2. Introduction to the main ideas

Over steep bathymetry, terrain-following coordinates give steeply sloping grid-cells (secs 1 & 7.2)



- The net horizontal pressure force on these cells can be calculated as the sum of the forces on the faces of the cell (Lin 1997).
- This is a good "conservative" framework.
- The force on the upper face segment Δx is - $p \; \Delta z$ (sec 2.1)



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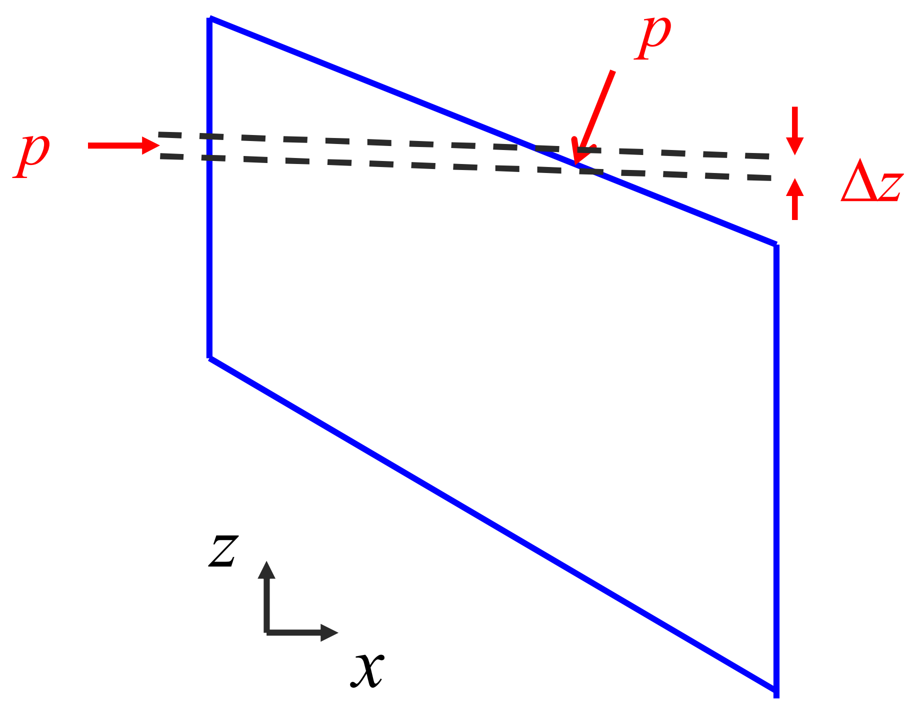
- When the tracers and pressure depend only on depth the net horizontal pressure force should be zero
- The net pressure force on the faces in any depth range is indeed zero (see previous slide)
- So if the pressure integrals are calculated accurately enough the "pressure gradient errors" will be small (sec 2.6)
- Second order accuracy is typically not good enough (secs 2.6 & 5)



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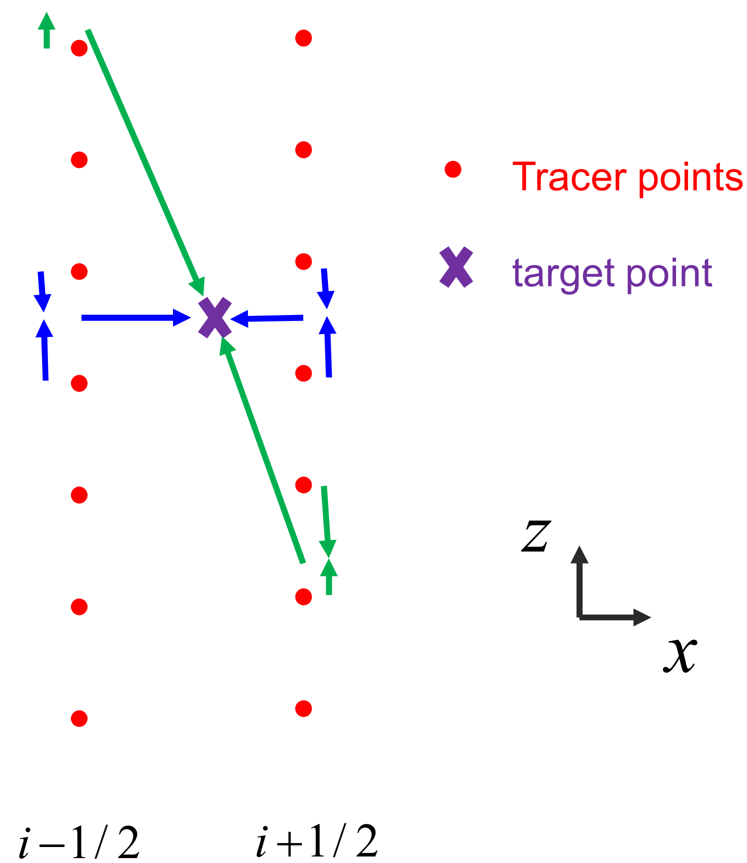
- Hydrostatic consistency problems (Haney 1991) occur if inappropriate stencils are used (sec 3.2)
- for example if interpolation is along constant s-surfaces (green lines) rather than constant z-surfaces (blue lines)
- These problems get worse as the model's vertical resolution improves
- Interpolation on constant z-surfaces needs care over bathymetry but is possible (sec 3.3)



The pressure integrals can be calculated very accurately by using local polynomials and Gaussian quadrature methods (Engwirda et al 2015) (secs 3.1 and 5.2)

For Lin's 2nd order accurate scheme the pressure forces do no net work (sec 2.5 and app A) and do not generate spurious vertically integrated circulations (sec 2.4)

Shchepetkin & McWilliams (2003) propose a somewhat similar naturally "conservative" framework based on the density Jacobian of Song (1998) (sec 4). They choose to interpolate along constant x and s-surfaces

We aim to systematically explore the range of possibilities (Jacobian & finite volume, different orders of accuracy and smoothing methods) (sec 6)

3. Some initial results (sec 5)

- We have obtained some first results for the net pressure forces on very steeply sloping cells for a standard static test case with $\rho = 6\exp(z/500)$ where ρ is the density (kgm^{-3}) and z is the height (m).

- We've looked at a single column of N cells. The ocean depth is $H_l = 1000$m on the lhs of the cell and H_r metres on the rhs.

- Representative results are presented for cells at mid-latitude with $\Delta x = 10$ km

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Geostrophic velocity errors (cm/s) due to inaccuracies in calculation of pressure forces as a function of the order of accuracy of the calculations

H_r	N	3rd	5th
800	10	0.59	0.06
400	10	2.9	0.11
800	50	$7\ 10^{-3}$	$1.3\ 10^{-5}$
400	50	2.4	$1.8\ 10^{-4}$

4. Open / unresolved issues (1)

- What coordinates should we allow (s or more general ALE) ?
- Should we take grid cell values to be volume averages rather than point values (sec 7.3) ?
- Should the T cells (rather than the u-cells) be simple quadrilaterals (sec 7.1 and app C) ?
- Should we take proper account of the y-variation in the shape of u-cells (sec 7.1 and app D) ? (This is possible in the finite volume approach but 1D line integrals become more expensive 2D surface integrals)
- How important is it to “conserve” energy in higher order formulations ? (app B)

4. Open / unresolved issues (2)

- What is the priority for choice of gradient limiters ? (sec 7.2)
- Will the roughness of the bathymetry undermine the higher order accuracy methods ?
- Should the smoothing of bathymetry be dependent on the scheme ?
    - With a hydrostatically consistent scheme the curvature of the bathymetry should be more important than the gradient
    - 2D spline fits to the bathymetry might then be a good choice

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5. Summary

- There are strong motivations to use terrain-following coordinates
- Pressure force errors can be greatly reduced by:
  - calculating the forces on the faces of the cells (or using line integrals)
  - using appropriate stencils for interpolation
  - interpolating and integrating with high order accuracy
- Next steps are to:
  - implement these ideas within NEMO
  - look at the time evolution of the errors

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References

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