# Adaptive vertical coordinates in GETM

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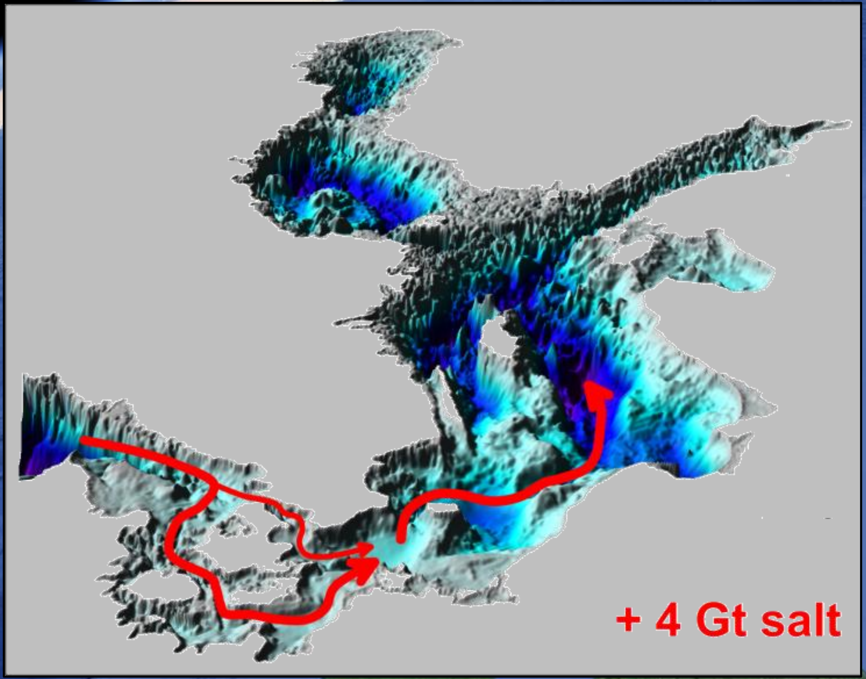
 Photography by Susanne Feistel, IOW

# The Baltic Sea as a natural laboratory





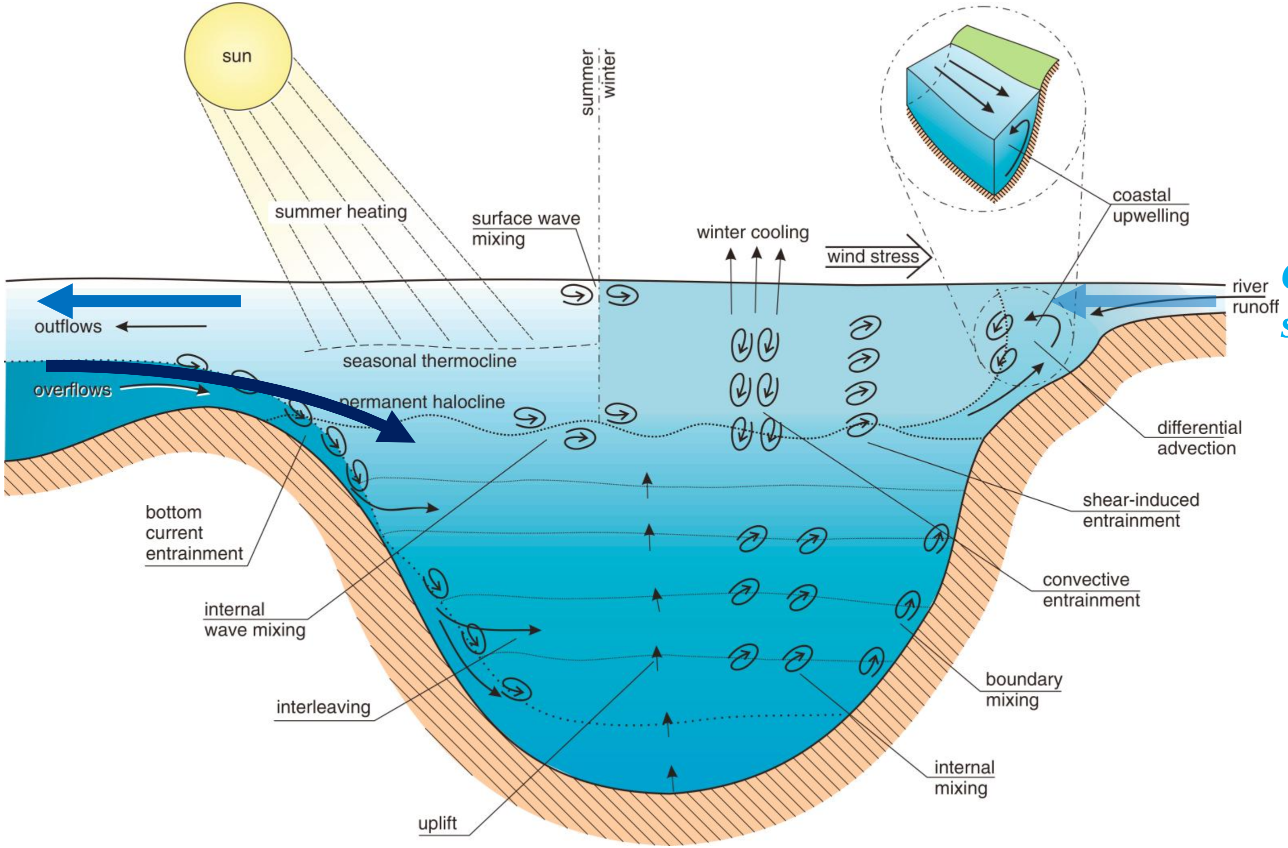
**Mohrholz et al. (2015) Fresh oxygen for the Baltic Sea — An exceptional saline inflow after a decade of stagnation. *JMS***

 Screenshot from google.com/maps

# The Baltic Sea as a natural laboratory - a nontidal mixing machine

$Q_{out} \approx 32 \times 10^3 m^3/s$
$s_{out} \approx 9$ g/kg

$Q_{in} \approx 16 \times 10^3 m^3/s$
$s_{in} \approx 17$ g/kg

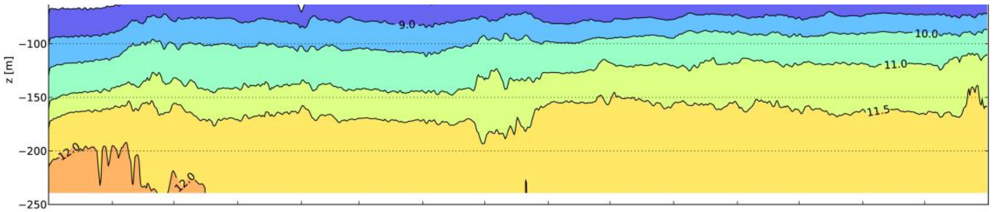



$Q_r \approx 16 \times 10^3 m^3/s$
$s_r = 0$ g/kg

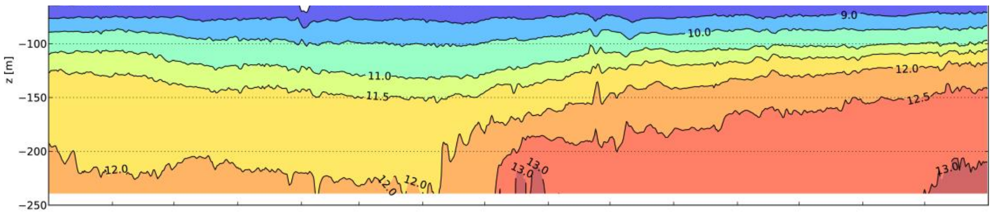
**Knudsen (1900) Ein hydrographischer Lehrsatz.** ***Annalen der Hydrography und Maritimen Meteorologie***
**Reissmann et al. (2009) Vertical mixing in the Baltic Sea and consequences for eutrophication - A review.** ***Progress in Oceanography***

# **The Baltic Sea as a natural laboratory** - a challenge for models

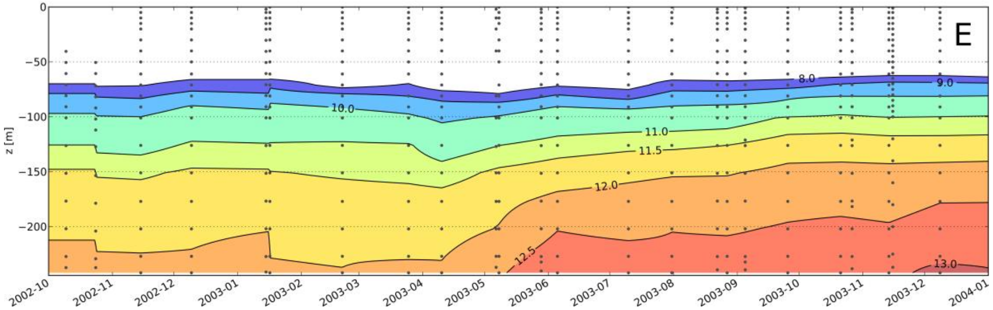
**Model with sigma coordinates**




**Model with adaptive coordinates**



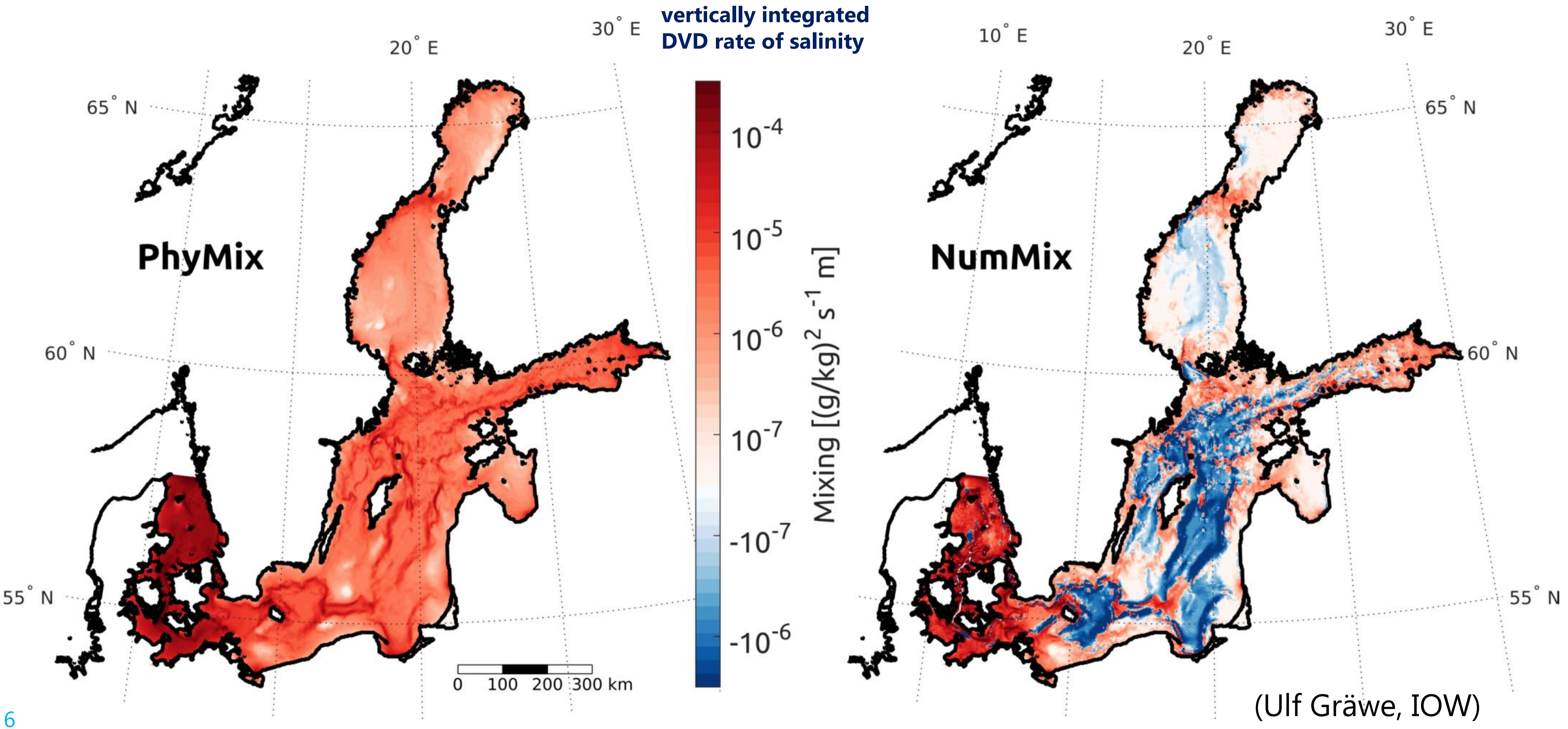

**Time series of measured salinity stratification at Gotland Deep**






**Hofmeister et al. (2011) Realistic modelling of the exceptional inflows into the central Baltic Sea in 2003 using terrain-following coordinates. *OCEMOD***

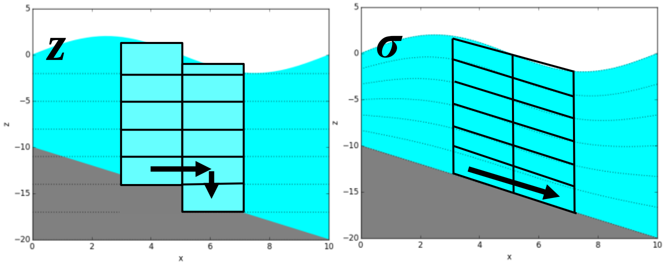
# Analysis of salinity mixing in the Baltic Sea




(Ulf Gräwe, IOW)

# Reduction of numerical mixing

➔ avoid advection?

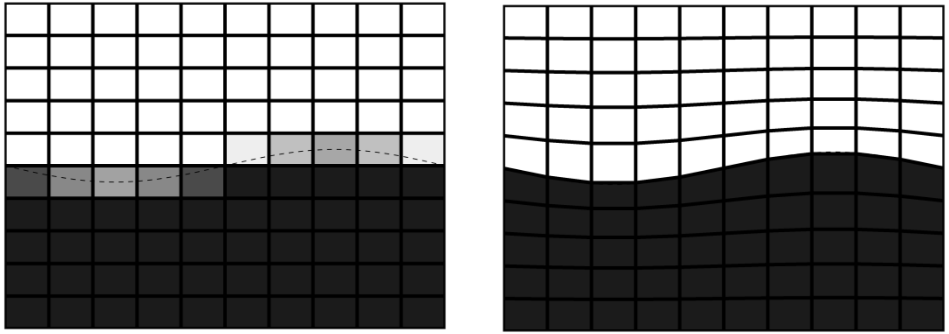



➔ reduce truncation errors of advection schemes

– more accurate advection schemes (higher-order, with advanced dissipation)

– refine spatial resolution

– align mesh to reduce tracer gradients across interfaces (e.g. isopycnal coordinates)



➔ move mesh to reduce flow relative to crossed interfaces

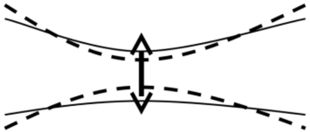
**Strategy: smart vertical meshes!**

# Smart vertical mesh

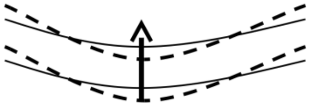
- ➔ reduce numerical mixing
- ➔ reduce pressure gradient errors (reduce spurious currents… mixing)
- ➔ more accurate isoneutral diffusion
- ➔ improved downslope flows
- ➔ reduced internal wave damping
- ➔ …

# Adaptive vertical coordinates in GETM

- ➔ Lagrangian tendency
- ➔ Horizontal diffusion of layer heights (reduce truncation errors in pressure gradient)



- ➔ Adjust for minimum thickness and rescale to match water depth
- ➔ Isopycnal tendency
- ➔ Horizontal diffusion of interface positions (decrease magnitude of pressure gradient terms)



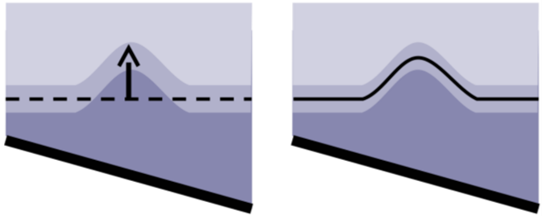
- ➔ Adjust for minimum thickness and rescale to match water depth
- ➔ Vertical diffusion equation for interface positions
- ➔ Adjust for minimum thickness and rescale to match water depth

# Isopycnal tendency

- ➔ isopycnal coordinates only Lagrangian without density mixing
- ➔ pure isopycnal coordinates ill-posed in well mixed areas
- ➔ difficulties for (non-linear) EOS with temperature and/or salinity evolution
- ➔ alternative: isopycnal tendency of interface positions

$$\hat{\rho} = \rho\left(\hat{z}\right) = \rho\left(z\right) + \left(\hat{z} - z\right)\frac{\partial \rho}{\partial z}$$

$$\hat{z} = z + \left(\frac{\partial \rho}{\partial z}\right)^{-1}\left(\hat{\rho} - \rho\left(z\right)\right)$$



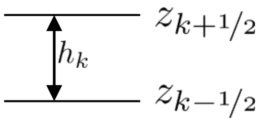
- ➔ clipping/stretching of layer heights to guarantee valid mesh

**Burchard & Beckers (2004) Non-uniform adaptive vertical grids in one-dimensional numerical ocean models. *OCEMOD***

**Hofmeister et al. (2010) Non-uniform adaptive vertical grids for 3D numerical ocean models. *OCEMOD***

# Adaptive refinement of vertical mesh resolution

➔ diffusion equation for vertical positions of layer interfaces




$$\frac{\partial z}{\partial t} - \frac{\partial}{\partial \kappa}\left\{D^{\text{mesh}}\frac{\partial z}{\partial \kappa}\right\} = 0$$

$$\frac{\partial z_{k+1/2}}{\partial t} - \left\{D_{k+1}^{\text{mesh}}\left(z_{k+3/2} - z_{k+1/2}\right) - D_k^{\text{mesh}}\left(z_{k+1/2} - z_{k-1/2}\right)\right\} = 0$$

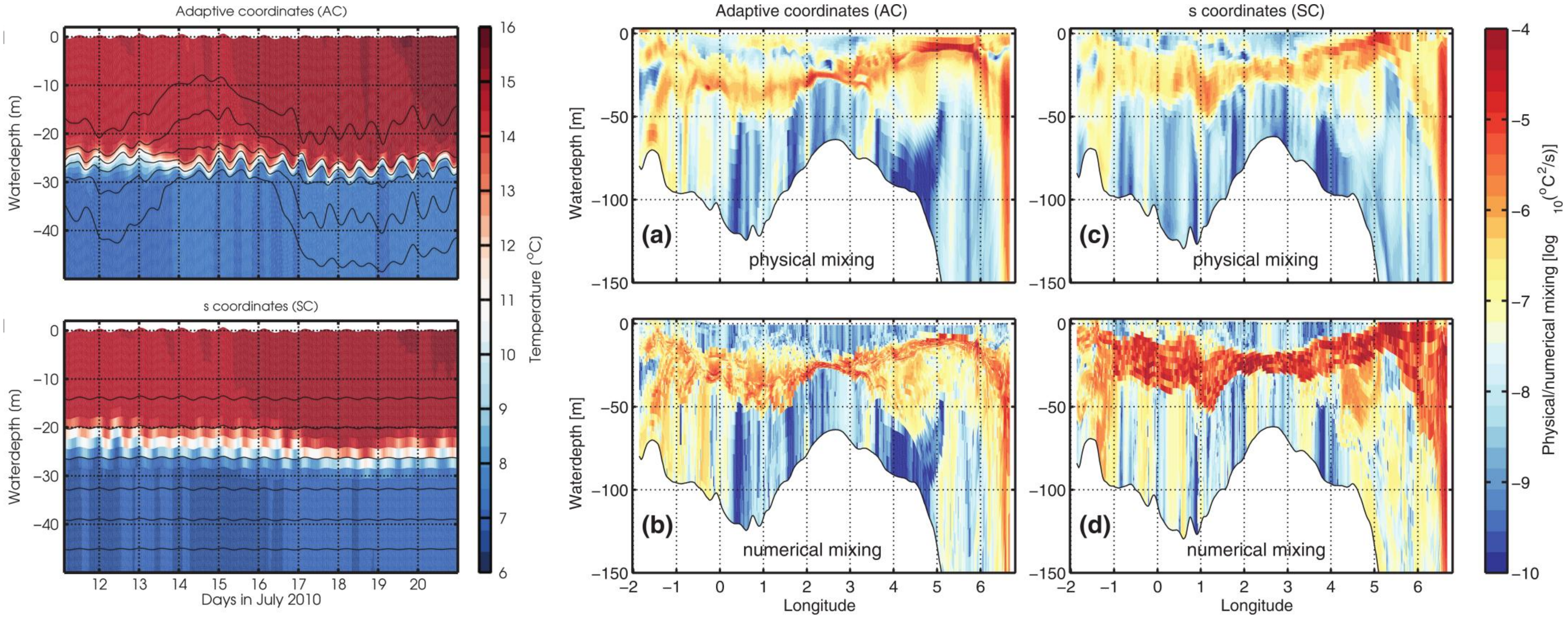
➔ $D_k^{\text{mesh}} h_k \approx D_{k+1}^{\text{mesh}} h_{k+1}$

➔ possible zooming towards:

– stratification, shear, boundaries


**Burchard & Beckers (2004) Non-uniform adaptive vertical grids in one-dimensional numerical ocean models. *OCEMOD***
**Hofmeister et al. (2010) Non-uniform adaptive vertical grids for 3D numerical ocean models. *OCEMOD***

# Adaptive refinement of vertical mesh resolution



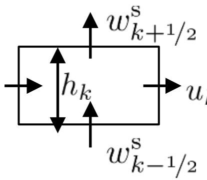



Gräwe et al. (2015) Advantages of vertically adaptive coordinates in numerical models of stratified shelf seas. *OCEMOD*

# Outlook: Arbitrary Lagrangian-Eulerian (ALE) coordinates

➔ layer-integrated advection equation

$$\partial_t \{h_k \varphi_k\} + \partial_x \{h_k u_k \varphi_k\} + w^{\mathrm{s}}_{k+1/2}\tilde{\varphi}_{k+1/2} - w^{\mathrm{s}}_{k-1/2}\tilde{\varphi}_{k-1/2} = 0$$

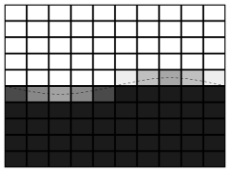



$w^{\mathrm{s}} = w - w^{\mathrm{mesh}}$

➔ layer-integrated continuity equation (volume conservation)

$$\partial_t h_k + \partial_x \{h_k u_k\} + w^{\mathrm{s}}_{k+1/2} - w^{\mathrm{s}}_{k-1/2} = 0$$

➔ *diagnostic* treatment: prescribe $h_k$ , calculate $w^{\mathrm{s}}_{k+1/2}$ $(w^{\mathrm{s}}_{1/2} = 0)$

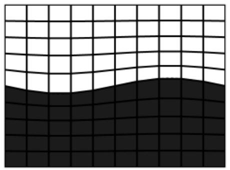


– Eulerian coordinates (*z*): $\partial_t h_k \equiv 0$

– sigma-coordinates: $h_k \equiv \triangle\sigma_k D$

**PROBLEM**: vertical advection can cause strong numerical mixing of tracers

➔ *prognostic* treatment: prescribe $w^{\mathrm{s}}$, calculate $h_k$



– Lagrangian coordinates: $w^{\mathrm{s}} \equiv 0$ (avoid vertical advection)

**PROBLEM**: prone to grid distortion in 3D

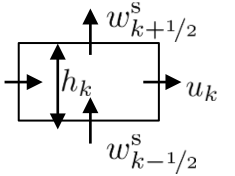


**Adcroft & Hallberg (2006) On methods for solving the oceanic equations of motion in generalized vertical coordinates. *OCEMOD***

# Outlook: Arbitrary Lagrangian-Eulerian (ALE) coordinates

➔ layer-integrated advection equation

$$\partial_t \{h_k \varphi_k\} + \partial_x \{h_k u_k \varphi_k\} + w^{\mathrm{s}}_{k+1/2} \tilde{\varphi}_{k+1/2} - w^{\mathrm{s}}_{k-1/2} \tilde{\varphi}_{k-1/2} = 0$$




$$w^{\mathrm{s}} = w - w^{\mathrm{mesh}}$$

➔ layer-integrated continuity equation (volume conservation)

$$\underbrace{\partial_t h_k + X_k}_{=0} + \underbrace{\partial_x \{h_k u_k\} - X_k + w^{\mathrm{s}}_{k+1/2} - w^{\mathrm{s}}_{k-1/2}}_{=0} = 0$$

➔ **How to deal with distortion of Lagrangian meshes?**

➔ **strategy 1**: fully Lagrangian + regrid/remap when mesh too distorted

➔ **strategy 2**: follow Lagrangian tendencies up to a desired level

- Lagrangian coordinates: $X \equiv \frac{\partial}{\partial x} \{hu\}$
- Eulerian coordinates ($z$): $X \equiv 0$
- sigma-coordinates: $X \equiv \frac{\partial}{\partial x} \{hU\}$
- $\tilde{z}$ -coordinates: $X \equiv \frac{\partial}{\partial x} \{hU\} + \frac{\partial}{\partial x} \{h \langle u - U \rangle_{\mathrm{HF}}\}$

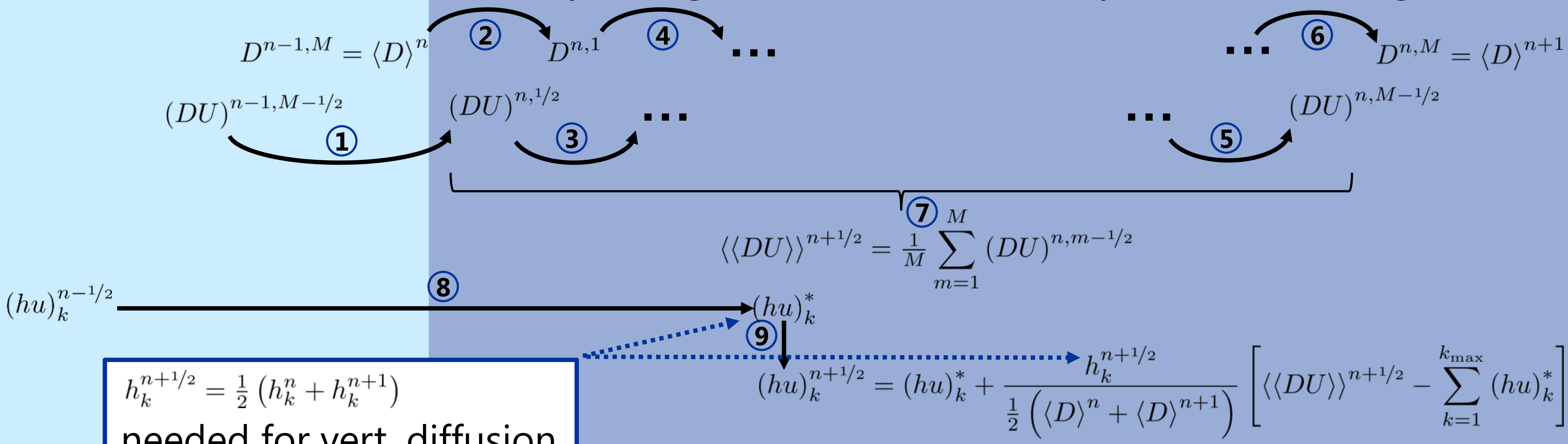
$$U = \frac{1}{D} \int u \mathrm{d}z$$



**Leclair & Madec (2011)$\tilde{z}$ -Coordinate, an Arbitrary Lagrangian-Eulerian coordinate separating high and low frequency motions. *OCEMOD***

# Outlook: Arbitrary Lagrangian-Eulerian (ALE) coordinates

- ➔ new implementation in GETM based on strategy 1
- ➔ challenge:
  consistent prognostic mesh in split-explicit mode-splitting models
  - – GETM needs final layer height to determine velocity at half-time stage




$$h_k^{n+1/2} = \frac{1}{2}\left(h_k^n + h_k^{n+1}\right)$$

needed for vert. diffusion and profile shifting (8, 9)